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SIMPLIFIED FORMS OF
PRELIMINARY TRAJECTORY CALCULATION
FOR GUN-LAUNCHED VEHICLES

by

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NOMENCLATURE

m	vehicle mass, slugs
g	acceleration of gravity, ft/sec^2
h	vehicle vertical altitude, ft
X	vehicle horizontal range, ft
θ	vehicle trajectory elevation angle from horizontal
t	time, secs
V	vehicle velocity, relative to earth, fps
V_j	rocket motor exhaust velocity, fps
$b = -\frac{dm}{dt}$	rocket motor burning rate, slugs/sec
$D = C_D\, q\, A$	aerodynamic drag on vehicle, lbs
$F = -V_j \frac{dm}{dt}$	rocket motor thrust on vehicle, lbs
A	vehicle cross-section reference area, sq ft
C_D	drag coefficient
$q = \frac{\gamma p}{2} M^2$	dynamic pressure, psf
γ	ratio of atmospheric specific heats at constant pressure and constant volume
$M = \frac{V}{a}$	vehicle Mach number
$p = \rho RT$	atmospheric pressure, psf
$a = \sqrt{\gamma RT}$	atmospheric sound speed, fps
T	atmospheric temperature, ^{o}Rankine
R	atmospheric gas constant, ft^2/sec^2 ^{o}Rankine
ρ	atmospheric density, slugs/ft^3
$N_R = \frac{Vd}{\nu}$	vehicle Reynolds number
d	vehicle base diameter, ft
ν	atmospheric kinematic viscosity, ft^2/sec

NOMENCLATURE (Cont'd)

K	constant in analytic approximation to drag coefficient
β	atmospheric lapse rate, ^{o}Rankine/ft
$\Delta(\ )$	increment in any quantity corresponding to increment in altitude
$(\ )_{n-1}$	quantity evaluated at beginning of increment in altitude
$(\ )_n$	quantity evaluated at end of increment in altitude
$\Delta I_n = \int_{h_{n-1}}^{h_n} (\frac{p}{a})dh$	aerodynamic drag integral, lb-sec/ft^2
$\overline{(\ )}_n$	arithmetic mean of quantities $(\ )_{n-1}$ and $(\ )_n$
$\Delta \mathcal{h}_n = \frac{2g\Delta h_n}{V_{n-1}^2}$	non-dimensional altitude increment
$\Delta \chi_n = \frac{2g\Delta X_n}{V_{n-1}^2}$	non-dimensional range increment
$\Delta \tau_n = \frac{g\,\Delta t_n}{V_{n-1}}$	non-dimensional time increment
$\Delta \mathcal{V}_n = \frac{\Delta V_n}{V_{n-1}}$	non-dimensional vehicle velocity increment
$\Delta \mathcal{V}_{g_n} = \frac{\Delta V_{g_n}}{V_{n-1}}$	non-dimensional vehicle gravity velocity decrement
$\Delta \mathcal{V}_{D_n} = \frac{\Delta V_{D_n}}{V_{n-1}}$	non-dimensional vehicle aerodynamic drag velocity decrement
$\Delta \mathcal{V}_{F_n} = \frac{\Delta V_{F_n}}{V_{n-1}}$	non-dimensional vehicle thrust velocity increment

NOMENCLATURE (Cont'd)

$\tilde{\theta}_n$ — elevation angle of chord line between points (n-1) and (n) on trajectory

$\eta = \dfrac{\Delta h_n}{\sin^2\theta_{n-1}}$ — non-dimensional altitude increment for universal zero-g trajectory

$\xi = \dfrac{\Delta x_n}{\sin 2\theta_{n-1}}$ — non-dimensional range increment for universal zero-g trajectory

or

$= \dfrac{g\,\Delta t_n}{V_{n-1}\sin\theta_{n-1}}$ — non-dimensional time increment for universal zero-g trajectory

$B = \dfrac{mg}{C_D A}$ — ballistic coefficient lbs/ft^2

$\alpha = \dfrac{F}{mg}$ — non-dimensional thrust parameter

$\Delta H = \dfrac{b_n}{m_{n-1}V_{j_n}\,\overline{\sin\theta_n}}\,\Delta h$ — non-dimensional altitude increment

$m = \dfrac{m}{m_{n-1}}$ — non-dimensional mass

$\mathcal{V}_j = \dfrac{V_j}{V_{n-1}}$ — non-dimensional rocket exhaust velocity

$\Delta J_n = \displaystyle\int_{h_{n-1}}^{h_n} \left(\frac{p}{a}\right)\frac{dh}{m}$ — aerodynamic drag integral, $lb\text{-}sec/ft^2$

c — coefficient in exponential atmosphere model, /kft

$\mathcal{f}_n = c - \dfrac{1.10\,b_n}{\mathcal{V}_{j_n}^{0.40}\,m_{n-1}V_{n-1}\,\overline{\sin\theta_n}}$ — parameter in evaluation of ΔJ_n

NOMENCLATURE (Cont'd)

$\varepsilon_{n-1} = \dfrac{V_{n-1}\, b_n}{g\, m_{n-1}}$	parameter in evaluation of $\Delta \mathcal{V}_n$
$\delta_n = \dfrac{\overline{\sin \theta_n}}{\varepsilon_{n-1}}$	parameter in evaluation of $\Delta \mathcal{V}_n$
U	absolute velocity of vehicle, fps
U_E	earth surface velocity at gun muzzle, fps
χ	angle between vectors $\vec{V}$ and $\vec{U}_E$, radians
ψ_v	azimuth angle of gun, radians
σ	path angle of trajectory (absolute) to local horizontal, radians
ψ_u	absolute azimuth angle, radians
r	radius from earth centre, ft
ϕ	angle of radius vector in trajectory plane, measured from perigee, radians
$C = r_{n-1} U_{n-1} \cos \sigma_{n-1}$	constant in tangential equation of motion, ft^2/sec
$\Delta h = \dfrac{\Delta h}{r}$	non-dimensional altitude increase
S,Q	parameters introduced in solutions of integrals
$\Delta \phi'$	range increment angle, radians

1.0 INTRODUCTION

The precise calculation of the motion of a gun-launched vehicle through and out of the atmosphere depends on a large number of parameters. These include the environmental parameters of atmospheric properties and gravity, which vary with altitude, and effects of the earth's rotation, which depend also on the launch direction. Then there are the vehicle parameters of mass, external size and shape, motion of control surfaces, specific impulse, mass ratio, and ignition and burning time of any rocket motors, and auxiliary or directional thrust systems. Finally there are the gun launch parameters of initial velocity and direction.

Inclusion of the accurate variation of these parameters along the trajectory leads to equations of motion which cannot be solved exactly. Consequently, various numerical computer programs have been devised, and these satisfactorily determine trajectories for given values of the parameters. In general they also can rapidly display the results of arbitrary variation of the parameters, so that optimum trajectories among those calculated can be selected.

This approach to trajectory calculation, while probably essential for accurate final calculations for a particular vehicle mission, suffers from two serious defects when applied to preliminary trajectory calculations, in which possible missions for existing or proposed vehicles are being considered. First, a computer may not be as readily available as the requirements for a set of rapid preliminary calculations would indicate. Second,

and more serious, as in all numerical programs the output of numbers has to be interpreted and analysed for trends in the light of the parameter input. There are no analytical forms linking input and output, from which trends could be discerned and predictions made, and as a result it is difficult to reach clearcut decisions about new vehicle missions.

It would therefore be useful if simplified forms of the governing equations could be developed which would retain the essential features of the exact equations and produce vehicle trajectories and other performance characteristics accurate to within 5 or even 10 percent, while providing straightforward analytical or graphical links between the various parameters and the resulting performance. That is the main purpose of the present report. An additional purpose is to provide, for reference, derivations of some of the important equations governing the motion of gun-launched vehicles.

2.0 FUNDAMENTAL ASSUMPTIONS

It will be assumed that the vehicles considered are aerodynamically symmetric with respect to their trajectory at all times, so that they experience no lift, merely drag, in their motion through the atmosphere. Any rocket thrust will be assumed to be constant in magnitude, and directed back along the trajectory, so that the vehicles are turned only by gravity. During motion of the vehicle either through the atmosphere or under the action of rocket thrust, or both, it will be assumed that the gravitational force is constant in magnitude and direction. The above assumptions will be accurate to within 3 percent for almost all cases.

2.1 BASIC EQUATION OF MOTION ALONG TRAJECTORY

In the light of the above assumptions, Figure 1 serves to define the equation of vehicle motion along its trajectory. Newton's Second Law of Motion requires 'Resultant External Force along Trajectory = Time Rate of Change of System Momentum along Trajectory'. The external forces are the tangential component of gravitational force mg sin θ and the aerodynamic drag D. The rocket motor thrust F is an internal force accounted for by the system momentum change. Consider the vehicle at time t when its mass is m and its velocity V. After an infinitesimal time increment Δt the rocket exhaust of constant velocity V_j has expelled mass $-\Delta m$ (the minus sign preserves the algebraic sign convention) and the velocity of the vehicle has increased to $V + \Delta V$. Thus Newton's Second Law becomes:

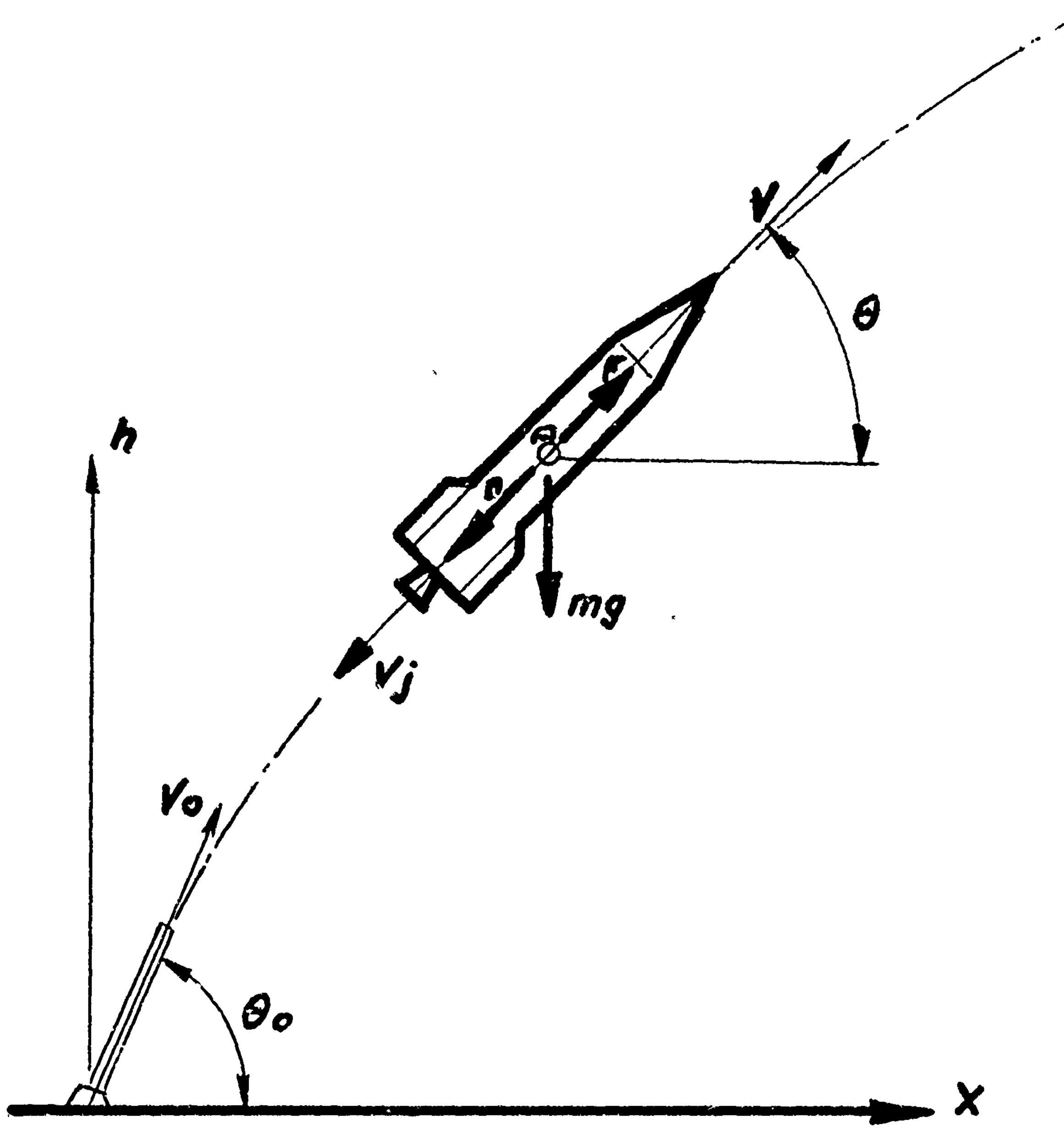


Figure 1 Vehicle Trajectory Parameters

$$-D - mg \sin\theta = \lim_{\Delta t \to 0} \frac{\left\{(m + \Delta m)(V + \Delta V) - \Delta m (V - V_j)\right\} - \left\{mV\right\}}{\Delta t}$$

$$= m \frac{dV}{dt} + V_j \frac{dm}{dt}$$

or

$$F - D - mg \sin\theta = m \frac{dV}{dt} \tag{2.1}$$

where $F = -V_j \frac{dm}{dt} = V_j b$, and b is the constant motor burning rate.

If Eq. (2.1) is multiplied through by $\frac{dt}{m}$ and then integrated from an initial state denoted $(\ \ )_{n-1}$ on the trajectory to a final state denoted $(\ \ )_n$ the result, using the fact that the rate of increase of altitude,

$$\frac{dh}{dt} = V \sin\theta \tag{2.2}$$

is

$$V_n = V_{n-1} + V_j \ln \frac{m_{n-1}}{m_n} - \int_{h_{n-1}}^{h_n} \frac{D}{mV \sin\theta}\, dh - g \int_{h_{n-1}}^{h_n} \frac{dh}{V} \tag{2.3}$$

or

$$\Delta V_n = \Delta V_{F_n} - \Delta V_{D_n} - \Delta V_{g_n}$$

In Eq. (2.3), the integrals for the velocity decrements from aerodynamic drag and gravity cannot be evaluated by quadratures without the introduction of further assumptions. The integral for ΔV_{D_n} is considered first.

2.2 AERODYNAMIC DRAG OF GUN-LAUNCHED SYSTEMS

Aerodynamic drag is conventionally expressed in terms of a drag coefficient C_D through the defining equation

$$D = C_D\ q\ A \tag{2.4}$$

where A = reference area = projected frontal area of body for a rocket or

gun-launched vehicle,

q = dynamic pressure.

In high speed aerodynamics the drag is caused by both the compressibility and friction of the air, and therefore the drag coefficient depends upon both Mach number M and Reynolds number N_R,

$$C_D = C_D\,(M,\ N_R) \qquad (2.5)$$

where $M = \frac{V}{a}$ = Vehicle Mach number

$N_R = \frac{Vd}{\nu}$ = Vehicle Reynolds number

a = local atmospheric speed of sound

d = vehicle base diameter

ν = local atmospheric kinematic viscosity.

It is convenient to express the dynamic pressure in terms of Mach number,

$$q = \frac{\gamma p}{2} M^2 \qquad (2.6)$$

where γ = ratio of atmospheric specific heats at constant pressure and constant volume

= 1.40, assumed constant

p = local atmospheric pressure.

The dependence of C_D on M and N_R is complex, even for simple vehicle shapes, and it has been common to make approximations in performance analysis. In the present analysis, the nature of gun-launching permits a very simple and useful approximation to be made. All gun-launched vehicles have initial

Mach numbers greater than 3, and even without rocket boost, vehicle Mach numbers will rarely fall much below 3 during motion through atmosphere dense enough to produce significant drag. Therefore it is not necessary to consider the drag variation of the vehicles in subsonic and transonic flight (as is required for conventional rocket-launched systems). Only the drag variation in supersonic and hypersonic motion need be considered and this is much simpler.

Over the range of velocities through the atmosphere experienced by a vehicle on a typical gun-launched mission, the variation of C_D with N_R will be quite small, and it is assumed henceforth that

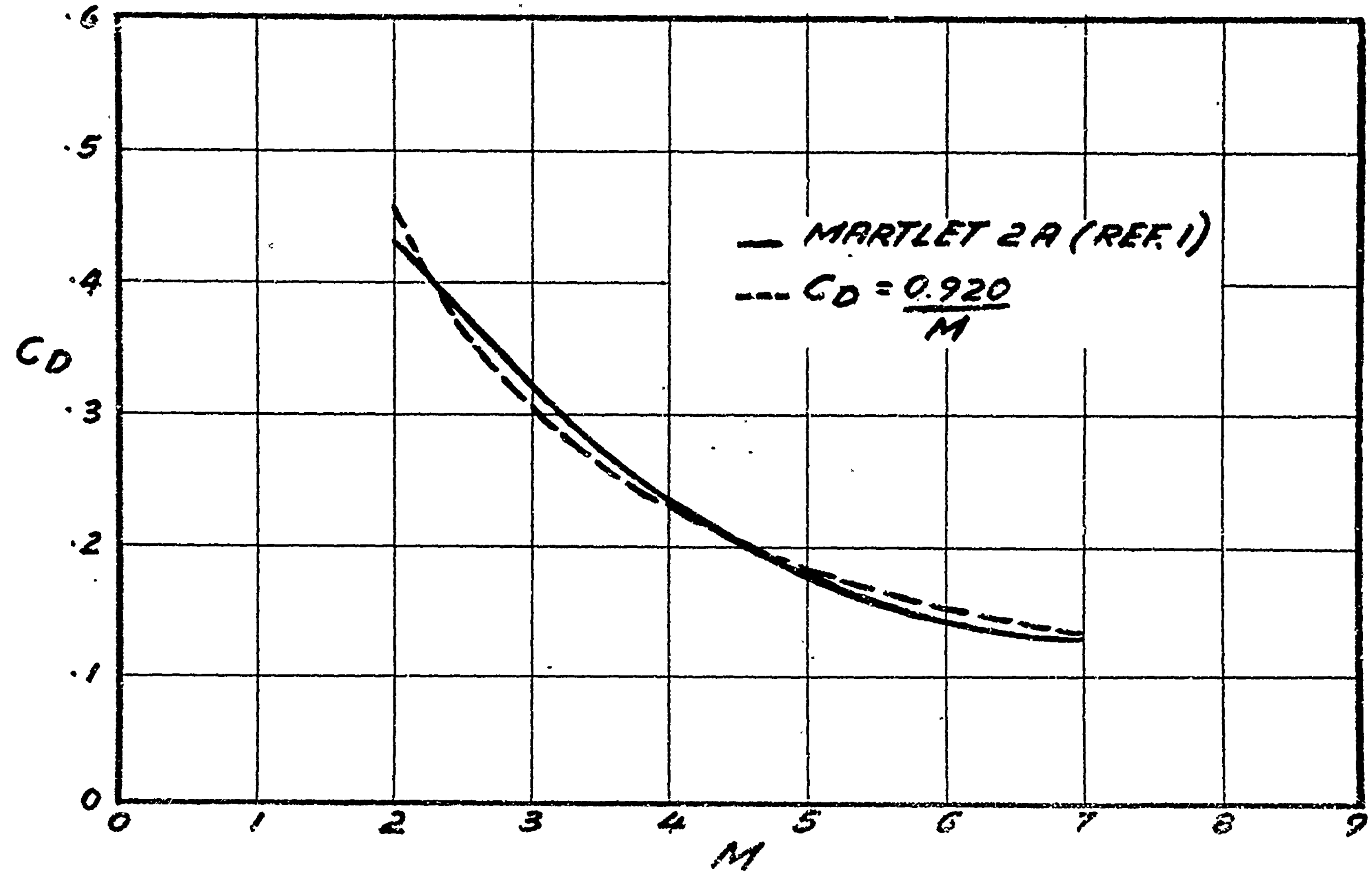
$$C_D = C_D (M) \qquad (2.7)$$

only for a particular vehicle. This variation can be approximated quite accurately in the supersonic and hypersonic range by the relation

$$C_D = \frac{K}{M} \qquad (2.8)$$

where K is a constant chosen to provide the best fit of Eq. (2.8) to the available data for the vehicle over the Mach number range of interest. In Figure 2 such a fit is shown for the Martlet 2A glide vehicle, the data being obtained from Reference 1. In the figure the maximum deviation of the approximate from the actual curve is 7.7 percent, and since it is clear that positive and negative deviations will tend to cancel in the drag integral of Eq. (2.3), and that in any case the actual values of C_D may not be accurately known at the time of the preliminary calculations for which this analysis is intended, the approximation of C_D by Eq. (2.8) is seen to be satisfactory.

Figure 2 Analytical Approximation to Drag Coefficient



Therefore, Eq. (2.4) becomes

$$D = \frac{K\gamma A}{2} \; p \; M \tag{2.9}$$

and the drag integral can be written

$$\Delta V_{D_n} = \frac{K\gamma A}{2} \int_{h_{n-1}}^{h_n} \left(\frac{p}{a}\right) \frac{dh}{m \sin \theta} \tag{2.10}$$

In the integrand in Eq. (2.10), (p/a) is a property of the local atmosphere, and is a function of h only for a given model of the atmosphere. As the vehicle climbs through the atmosphere after gun launch, sin θ is a slowly decreasing function of h which of course is unknown until the trajectory is determined. However, the initial value is known, and because of the slow decrease with increasing h, a sufficiently accurate average value $\overline{\sin \theta_n}$ can be determined from the equivalent zero-drag trajectory between the same altitude limits. The drag integral can then be written

$$\Delta V_{D_n} = \frac{K\gamma A}{2\,\overline{\sin \theta_n}} \int_{h_{n-1}}^{h_n} \left(\frac{p}{a}\right) \frac{dh}{m} \tag{2.11}$$

For vehicles without rocket motors, or for glide sections of any gun-launched vehicle trajectory, m is constant and Eq. (2.11) can be evaluated directly. During motion through the atmosphere under rocket thrust, m decreases at a constant time rate, and again the equivalent zero-drag trajectory can be used to approximate m as an integrable function of h between the same altitude limits.

The significant observation to be made from Eq. (2.11) is that,

within the accuracy of Eq. (2.8), the decrement in velocity experienced by a supersonic or hypersonic gun-launched vehicle due to atmospheric drag is <u>not</u> increased by increasing the launch velocity of the vehicle, as one might expect intuitively.

On the contrary, for a vehicle without rocket motor, or during a glide section of any vehicle trajectory, the drag velocity decrement is nearly independent of the vehicle velocity, which enters only through its rather small effect on the value of $\overline{\sin\theta_n}$ in Eq. (2.11), and here the effect of increased launch velocity is to increase $\overline{\sin\theta_n}$ and thus <u>reduce</u> the drag decrement.

For a rocket-powered section of vehicle trajectory with a motor of given burning rate, the effect of increased launch velocity is to reduce the decrease of vehicle mass m in passing through a given altitude increment, and thus again <u>reduce</u> the drag velocity decrement.

In general, then, Eq. (2.11) shows that for any gun-launched vehicle, the <u>higher</u> the launch velocity, the <u>lower</u> will be the resulting decrement in velocity due to aerodynamic drag.

2.3 THE MODEL ATMOSPHERE

Before Eq. (2.11) can be integrated, the dependence of (p/a) on h for the atmosphere under consideration must be put in suitable analytic form. Here the model used assumes a linear variation of temperature T with altitude,

$$T = T_{n-1} - \beta (h - h_{n-1}) \tag{2.12}$$

where

β = lapse rate = constant.

The air is assumed to obey the equation of state of a perfect gas

$$p = \rho RT \tag{2.13}$$

where ρ = air density

R = specific gas constant

= 1716 $ft^2/(sec^2 . \,^oRankine)^{-1}$.

The additional relation needed is the vertical equilibrium equation for the static atmosphere

$$\frac{dp}{dh} = - \rho g \tag{2.14}$$

If ρ and T are eliminated from Eqs. (2.12), (2.13), and (2.14) the result is

$$\frac{dp}{dh} = - \frac{pg}{R \left\{ T_{n-1} - \beta (h - h_{n-1}) \right\}}$$

or

$$\int_{p_{n-1}}^{p} \frac{dp}{p} = - \frac{g}{R} \int_{h_{n-1}}^{h} \frac{dh}{\left\{ T_{n-1} - \beta (h - h_{n-1}) \right\}}$$

This equation is integrated directly to produce the pressure-altitude relation

$$\frac{p}{p_{n-1}} = \left\{ \frac{T_{n-1} - \beta (h - h_{n-1})}{T_{n-1}} \right\}^{\frac{g}{\beta R}} \tag{2.15}$$

The speed of sound in a perfect gas is given by

$$a = \sqrt{\gamma RT} \tag{2.16}$$

and so, using Eq. (2.12),

$$a = \sqrt{\gamma R \left\{ T_{n-1} - \beta (h - h_{n-1}) \right\}} \tag{2.17}$$

Therefore, the required function for (p/a) is, with a little rearrangement

$$\left(\frac{p}{a}\right) = \left(\frac{p_{n-1}}{a_{n-1}}\right) \left[\frac{T_{n-1} - \beta (h - h_{n-1})}{T_{n-1}}\right]^{\frac{g}{\beta R} - \frac{1}{2}} \tag{2.18}$$

For a particular atmosphere, β is chosen to fit the actual temperature variation within given altitude limits. In this report, the reference atmosphere is the Cape Kennedy Standard Atmosphere from Reference 2, and a very good fit is obtained using 3 values of β, as follows:

$0 < h < 50.8$ kft	$\beta_1 = 3.54\,^{\circ}R/\text{kft}$
$50.8 < h < 162.4$ kft	$\beta_2 = -1.235\,^{\circ}R/\text{kft}$
$162.4 < h < 270.6$ kft	$\beta_3 = 1.574\,^{\circ}R/\text{kft}$

Figures 3 and 4 show the close agreement between the pressure and sound speed from Reference 2 and the values given by Eqs. (2.15) and (2.17), using the above values of β. For this atmosphere, at sea level (h = 0):

$$p_o = 2125 \text{ psf}, \; a_o = 1138 \text{ fps}, \; g = 32.15 \text{ ft/sec}^2.$$

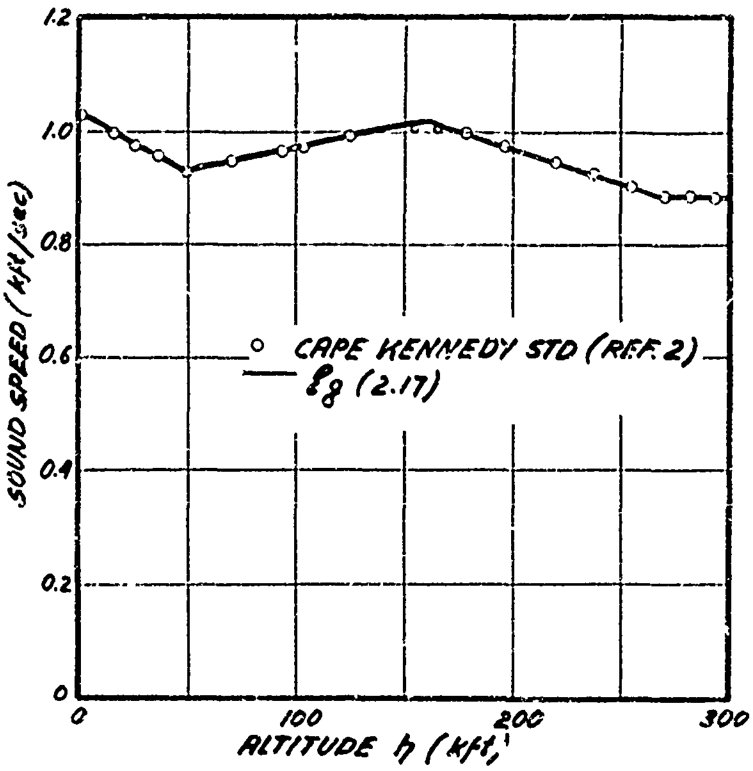


Figure 4 Atmospheric Sound Speed vs Altitude

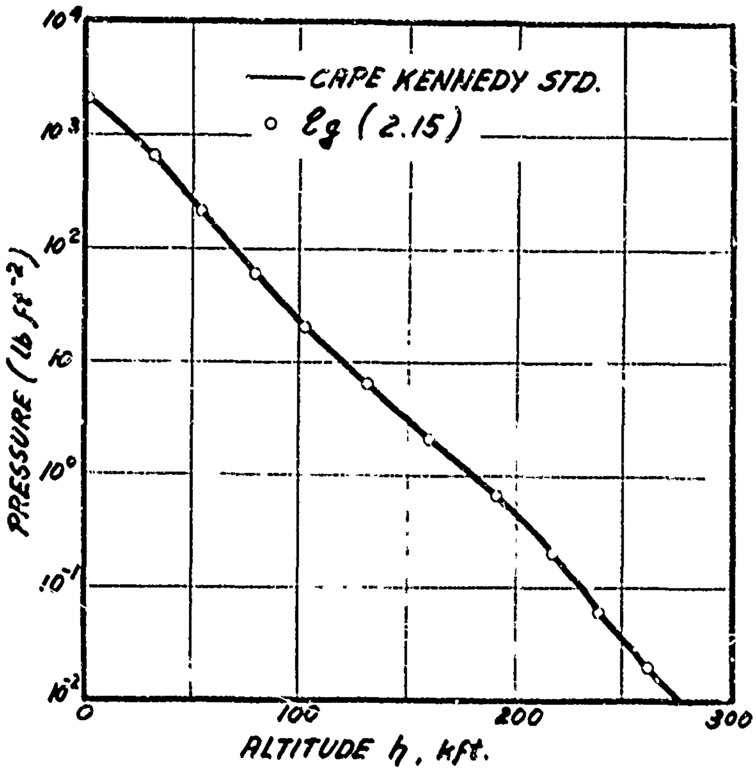


Figure 3 Atmospheric Pressure vs. Altitude

3.0 GLIDE TRAJECTORIES

When a gun-launched vehicle is not moving under rocket motor thrust, mass m is constant, and Eq. (2.11) becomes

$$\Delta V_{D_n} = \frac{K\gamma A}{2m \sin\theta_n} \int_{h_{n-1}}^{h_n} \left(\frac{p}{a}\right) dh = \frac{K\gamma A}{2m \sin\theta_n} \Delta I_n \qquad (3.1)$$

The integral ΔI_n is evaluated using Eq. (2.18) and the solution of Eq. (2.3) can then be considered.

3.1 EVALUATION OF DRAG INTEGRAL

$$\Delta I_n \equiv \left(\frac{p_{n-1}}{a_{n-1}}\right) \frac{1}{T_{n-1}^{\frac{g}{\beta R}-\frac{1}{2}}} \int_{h_{n-1}}^{h_n} \left\{T_{n-1} - \beta(h - h_{n-1})\right\}^{\frac{g}{\beta R}-\frac{1}{2}} dh$$

$$= -\left(\frac{p_{n-1}}{a_{n-1}}\right) \frac{1}{\beta T_{n-1}^{\frac{g}{\beta R}-\frac{1}{2}}} \frac{1}{\left(\frac{g}{\beta R}+\frac{1}{2}\right)} \left[\left\{T_{n-1} - \beta(h_n - h_{n-1})\right\}^{\frac{g}{\beta R}+\frac{1}{2}} - T_{n-1}^{\frac{g}{\beta R}+\frac{1}{2}}\right]$$

$$= \left[\frac{p_{n-1}a_{n-1} - p_n a_n}{\gamma g + \frac{\gamma R \beta}{2}}\right] \text{ lb-sec/ft}^2\text{; using eqs. (2.15) and (2.17).} \qquad (3.2)$$

Using the parameters for the model of the Cape Kennedy Standard Atmosphere, this function has been calculated for increments of altitude from sea level until incremental values of the function become negligible. The results are given in Table 1, and the total function I_n from that Table is plotted in Figure 5. It can be seen that the drag decrement in vehicle velocity becomes negligible above 160 kft.

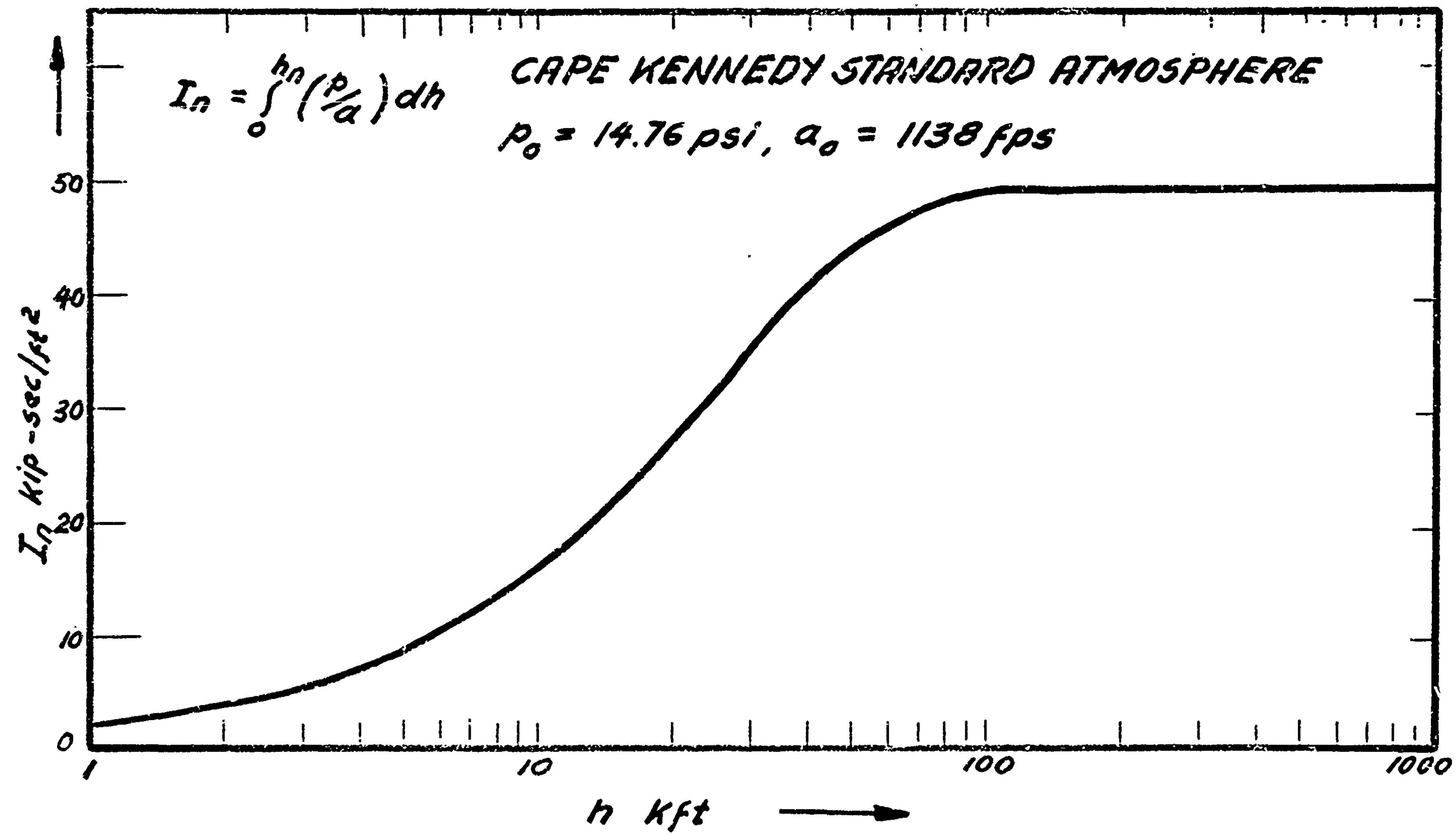
TABLE 1

Drag Decrement Integrals ΔI_n and I_n

No.	h kft	ΔI_n lb-sec/ft^2	I_n lb-sec/ft^2	No.	h kft	ΔI_n lb-sec/ft^2	I_n lb-sec/ft^2
0	0	0	0	14	65.6	2740	47100
1	1.00	2080	2080	15	82.0	1370	48500
2	2.00	1950	4030	16	98.4	630	49100
3	3.28	1810	5840	17	114.8	288	49400
4	6.56	5420	11260	18	131.2	141	49500
5	9.84	4660	15920	19	147.6	67	49600
6	13.12	4240	20160	20	162.4	34	49700
7	16.40	3760	23920	21	180.5	22	
8	19.68	3200	27120	22	196.8	11	
9	26.24	5420	32540	23	213.2	6	
10	32.80	4650	37190	24	229.6	3	
11	39.36	3300	40490	25	246.0	1	
12	45.92	2520	43010	26	262.4	1	
13	50.84	1390	44400	27	270.6	0	

TOTAL 49,700 lb-sec/ft^2

Figure 5 Total Drag Decrement Integral



3.2 ZERO-DRAG GLIDE TRAJECTORIES

The equations of motion for a vehicle moving without rocket thrust or atmospheric drag under constant g are very simple. They are useful in the present analysis both in providing a basis for approximating certain functions in Eq. (2.3), and in themselves as giving sufficiently accurate results for the upper parts of vehicle trajectories high enough to neglect aerodynamic drag but not so high as to require the inclusion of g variations.

The first relation needed is obtained directly from the conservation of mechanical potential and kinetic energy:

$$h_n - h_{n-1} = \frac{V_{n-1}^2 - V_n^2}{2g} \tag{3.3}$$

The other relations needed are obtained by manipulating the equations of vehicle motion in the X- and h- directions (see Figure 1), and are derived in Appendix A:

$$\Delta\mathcal{h}_n = \tan\theta_{n-1}\Delta\mathcal{X}_n - \frac{1}{4}\sec^2\theta_{n-1}\Delta\mathcal{X}^2_n \tag{3.4}$$

$$\sin\theta_n = \sqrt{\frac{\sin^2\theta_{n-1} - \Delta\mathcal{h}_n}{1 - \Delta\mathcal{h}_n}} = \sqrt{1 - \frac{\cos^2\theta_{n-1}}{1 - \Delta\mathcal{h}_n}} \tag{3.5}$$

or

$$\cos\theta_n = \frac{\cos\theta_{n-1}}{\sqrt{1 - \Delta\mathcal{h}_n}} \tag{3.6}$$

where

$$\Delta\mathcal{h}_n = \frac{2g}{V^2_{n-1}}(h_n - h_{n-1}),\quad \Delta\mathcal{X}_n = \frac{2g}{V^2_{n-1}}(X_n - X_{n-1}) \tag{3.7}$$

3.3 APPROXIMATIONS FOR GLIDE TRAJECTORIES WITH DRAG

For a vehicle moving without rocket thrust, Eq. (2.3) can be written, using Eq. (3.1):

$$\Delta V_n = - \Delta V_{D_n} - \Delta V_{g_n}$$

$$= \frac{K \gamma A}{2m \, \overline{\sin \theta_n}} \Delta I_n - g \int_{h_{n-1}}^{h_n} \frac{dh}{V} \qquad (3.8)$$

If there were no aerodynamic drag, Eq. (3.3) could be used, and written in the form showing the velocity decrement caused by gravity:

$$V_n - V_{n-1} = \frac{2g \, (h_n - h_{n-1})}{V_n + V_{n-1}}$$

or

$$\Delta V_n = - \frac{g \Delta h_n}{\overline{V}_n} = \Delta V_{g_n} \qquad (3.9)$$

where

$$\overline{V}_n = \tfrac{1}{2} (V_{n-1} + V_n) = V_{n-1} + \tfrac{1}{2} \Delta V_n \qquad (3.10)$$

It is now assumed that ΔV_{g_n} in Eq. (3.8) can be approximated by an expression of the form of Eq. (3.9), with $\overline{V}_n$ the arithmetic mean of the <u>actual</u> V_{n-1} and V_n:

$$\Delta V_n = - \Delta V_{D_n} - \frac{g \Delta h_n}{\overline{V}_n}$$

or, multiplying through by $\overline{V}_n$ and using Eq. (3.10):

$$\tfrac{1}{2} \Delta V_n^{\,2} + (V_{n-1} + \tfrac{1}{2} \Delta V_{D_n}) \Delta V_n + (V_{n-1} \Delta V_{D_n} + g \Delta h_n) = 0.$$

Solving for ΔV_n, and dividing through by V_{n-1}:

$$\frac{\Delta V_n}{V_{n-1}} = -\left(1 + \tfrac{1}{2}\frac{\Delta V_{D_n}}{V_{n-1}}\right) + \sqrt{\left(1 + \tfrac{1}{2}\frac{\Delta V_{D_n}}{V_{n-1}}\right)^2 - 2\frac{\Delta V_{D_n}}{V_{n-1}} - \Delta h_n}$$

or
$$\Delta v_n = -\left(1 + \tfrac{1}{2}\Delta v_{D_n}\right) + \sqrt{\left(1 - \tfrac{1}{2}\Delta v_{D_n}\right)^2 - \Delta h_n} \qquad (3.11)$$

where
$$\Delta v_n = \frac{\Delta V_n}{V_{n-1}}, \quad \Delta v_{D_n} = \frac{\Delta V_{D_n}}{V_{n-1}} \qquad (3.12)$$

Before Eq. (3.11) car be used to calculate velocity decrements along a glide trajectory, a method of determining $\overline{\sin\theta}_n$ in the expression for ΔV_{D_n} must be given. It can be shown that

$$\frac{1}{\overline{\sin\theta}_n}\int_{h_{n-1}}^{h_n} dh$$

is a good approximation to

$$\int_{h_{n-1}}^{h_n} \frac{dh}{\sin\theta}$$

where
$$\overline{\sin\theta}_n = \tfrac{1}{2}\left(\sin\theta_{n-1} + \sin\theta_n\right) \qquad (3.13)$$

and $\sin\theta_n$ is given by Eq. (3.5).

For example, with $\sin\theta_{n-1} = 0.50$ and $\sin\theta_n = 0.25$, the approximate formula is only 1% higher than the exact integral.

Therefore $\overline{\sin \theta_n}$ as defined by Eq. (3.13) is used in Eq. (3.1). In the calculation procedure next to be described, the fundamental independent variable is the altitude increment Δh_n. Calculation of the vehicle trajectory requires corresponding values of the horizontal range increment ΔX_n, shown in Figure 6. Since

$$\Delta X_n = \cot \tilde{\theta}_n \, \Delta h_n$$

and $\tilde{\theta}_n$ will not be appreciably different from $\bar{\theta}_n$ as given by Eq. (3.13) it is assumed that a satisfactory approximation is

$$\Delta X_n = \cot \bar{\theta}_n \, \Delta h_n \qquad (3.14)$$

In the same way it is assumed that the time increment Δt_n is given by

$$\Delta t_n = \frac{1}{\bar{V}_n \, \overline{\sin \theta_n}} \, \Delta h_n \qquad (3.15)$$

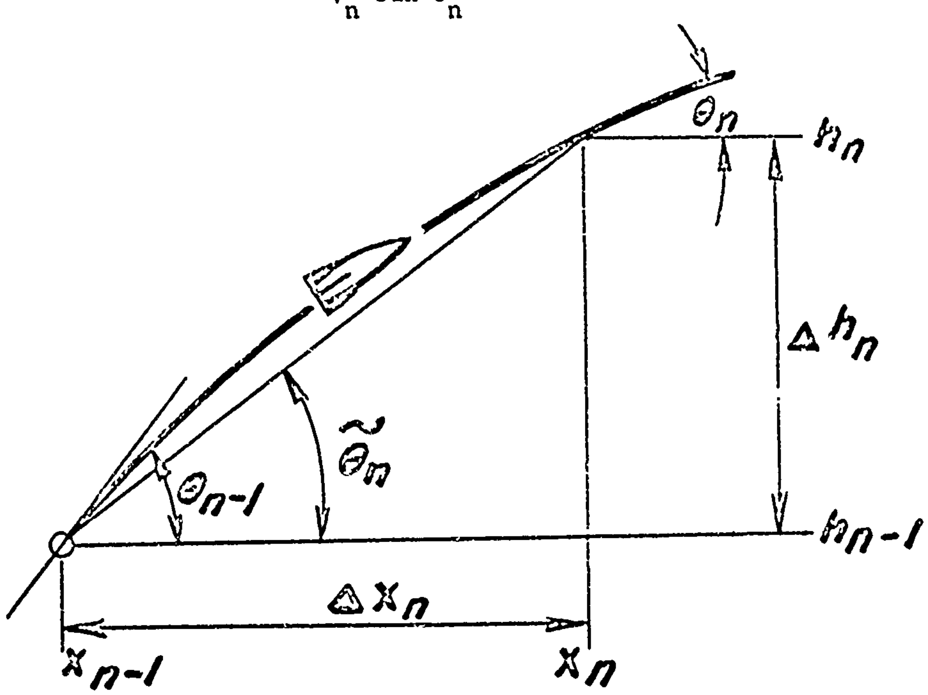


Figure 6
Basis of Horizontal Range Increment Approximation

3.4 CALCULATION PROCEDURE FOR CONSTANT-g GLIDE TRAJECTORIES

For a given vehicle launched under given conditions in a given model atmosphere, the following parameters are known:

$$g, m, A, K, \gamma, V_o, \theta_o$$

The factor $\frac{K\gamma A}{2m}$ in ΔV_{D_n} can therefore be calculated. The drag integral ΔI_n can be determined for each altitude increment Δh_n from Figure 5 or Table 1. The altitude to which the aerodynamic drag is significant is broken down into a few increments, the number increasing with the accuracy desired. Thus four increments should be sufficient for most preliminary calculations, while one may be enough for a rough calculation.

The procedure is then to use the equations of § 3.2 and 3.3 to determine the velocity decrement, horizontal range increment, time increment, and final elevation angle for each altitude increment. Above the final atmospheric altitude increment, the equations of § 3.2 and Appendix A are used to calculate the remainder of the trajectory to its apogee. Alternatively, Eq. (3.4) for the parabolic zero-drag trajectory can be put in the universal form:

$$\eta = 2\xi - \xi^2 \tag{3.16}$$

where ξ and η are defined in Appendix A. Eq. (3.16) is plotted in Figure 7 and all zero-drag, constant-g glide trajectories can be determined from it.

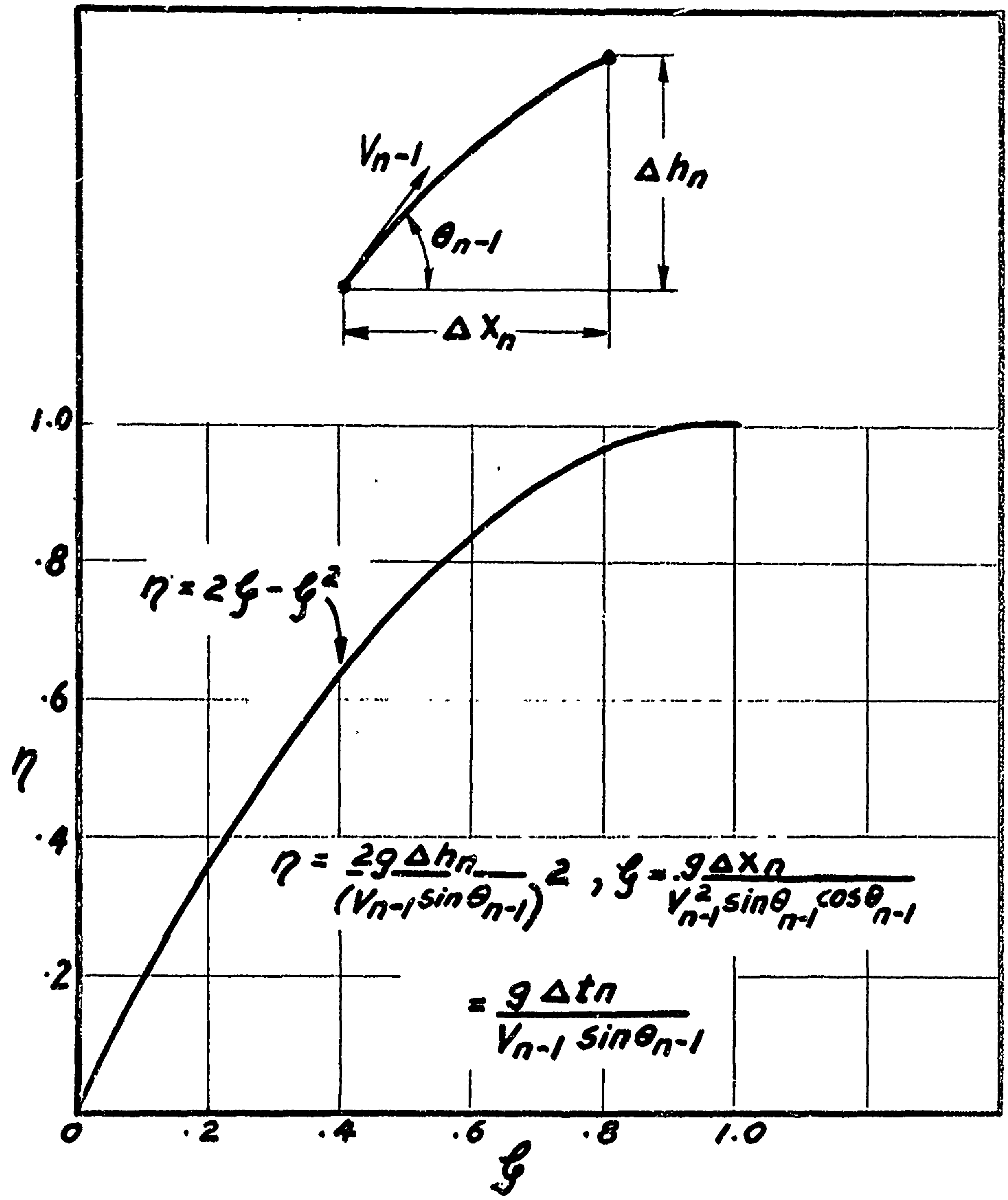


Figure 7 Universal Zero-Drag Constant-g Trajectory

In greater detail, the procedure for the atmospheric part of the trajectory, given h_{n-1}, Δh_n, V_{n-1}, θ_{n-1}, is to calculate $\Delta \mathcal{h}_n$ from Eq. (3.7) and use this to calculate $\sin \theta_n$ from Eq. (3.5) or $\cos\theta_n$ from Eq. (3.6). (The latter is more convenient for values of θ_{n-1} near 90°.) This determines the new elevation angle θ_n. Next $\overline{\sin \theta}_n$ is found from Eq. (3.13), giving $\overline{\theta}_n$. With ΔI_n determined for the given Δh_n from Figure 5 or Table 1, ΔV_{D_n} is calculated from Eq. (3.8), and $\Delta \upsilon_{D_n}$ from Eq. (3.12). Now $\Delta \upsilon_n$ can be calculated from Eq. (3.11), and this gives the velocity change ΔV_n from Eq. (3.12). The range increment ΔX_n is calculated from Eq. (3.14) and this gives the new X_n corresponding to h_n. The time increment Δt_n is calculated from Eq. (3.15), giving the new t_n. The new velocity V_n is given by adding ΔV_n to V_{n-1}. The procedure is then repeated for the next altitude increment.

Figure 8 gives the application of the procedure to the trajectory of the Martlet 2A, IOWA shot from the Barbados 16" gun on March 25, 1965. Results of calculations with both four and one atmospheric altitude increments are compared with radar-determined trajectories of the actual shot from Reference 1, and with the standard HARP trajectory from Reference 3. Curves are given for vehicle weights of both 170 lbs and 180 lbs, since the actual vehicle weighed 170 lbs, while the HARP trajectory was calculated for 180 lbs. Figures 9 and 10 show the variation of vehicle velocity and elapsed time with altitude, up to the apogee, for the 180 lb vehicle as calculated by the present method with both four and one atmospheric steps, and by the standard HARP computer program. Calculations for Figures 8, 9 and 10 are tabulated in Appendix B. In Figure 11, the 180 lb trajectories are compared again for greater clarity.

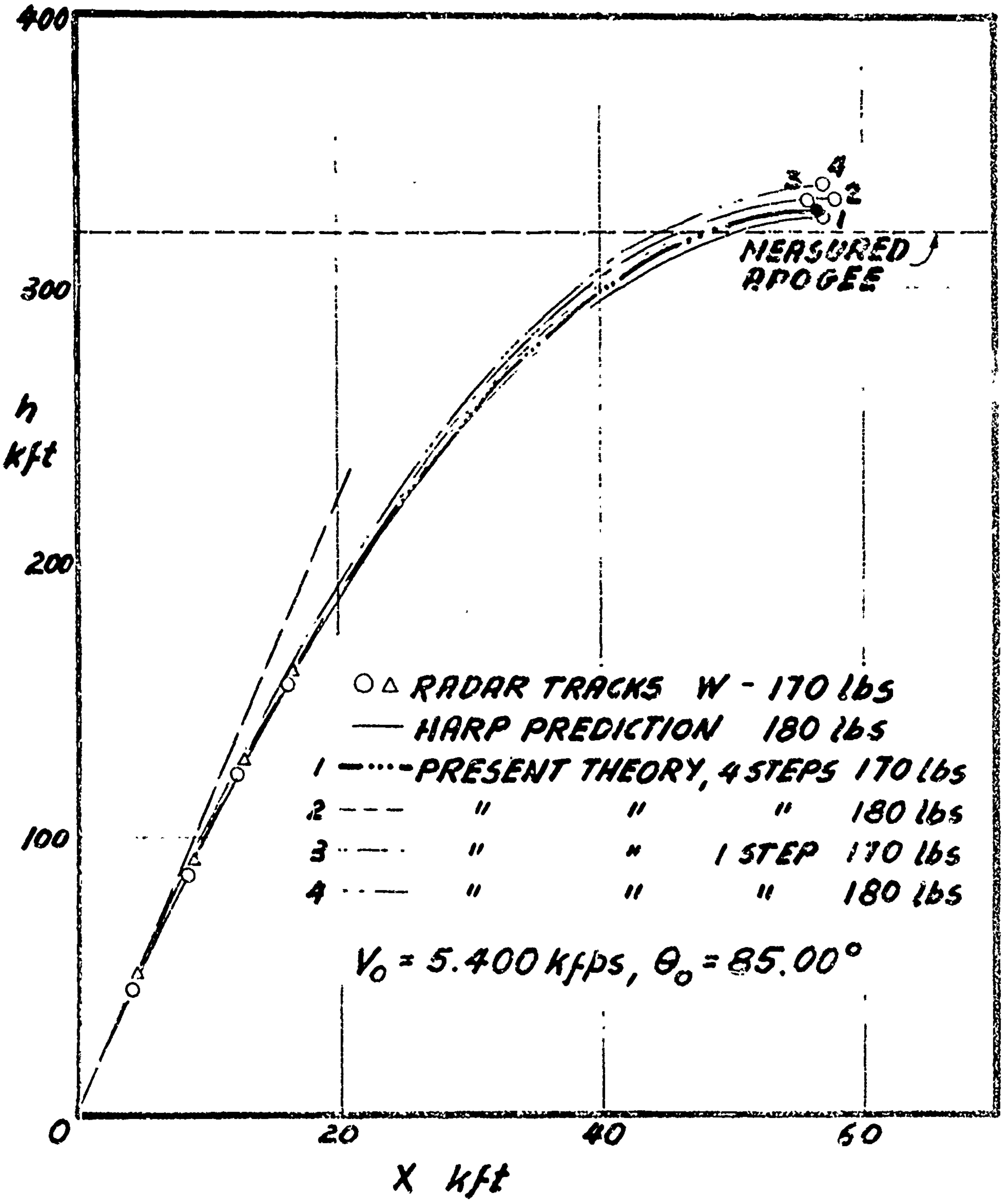


Figure 8 Martlet 2A Shot IOWA Trajectory

Figure 9 Velocity vs Altitude for Martlet 2A-Shot

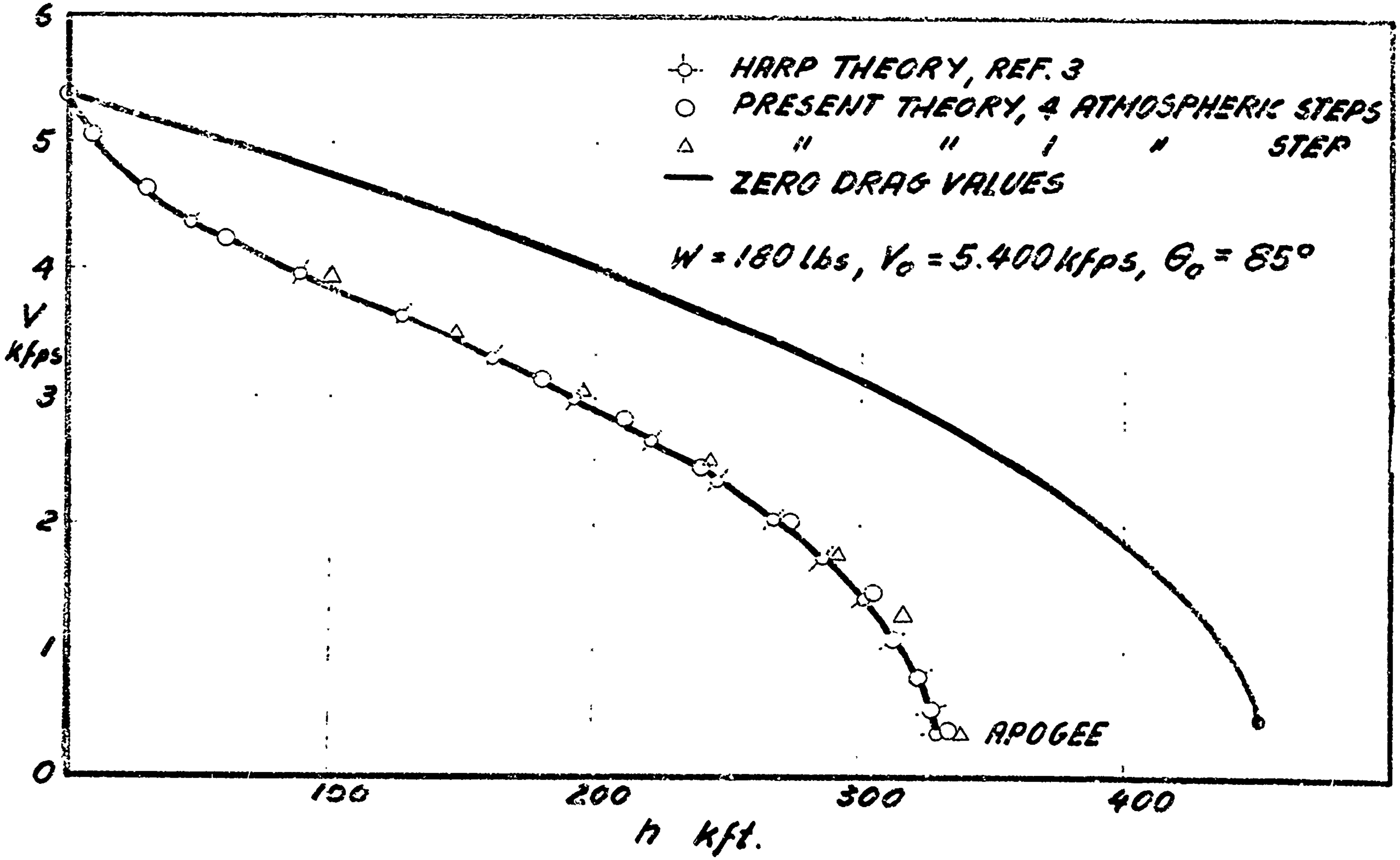
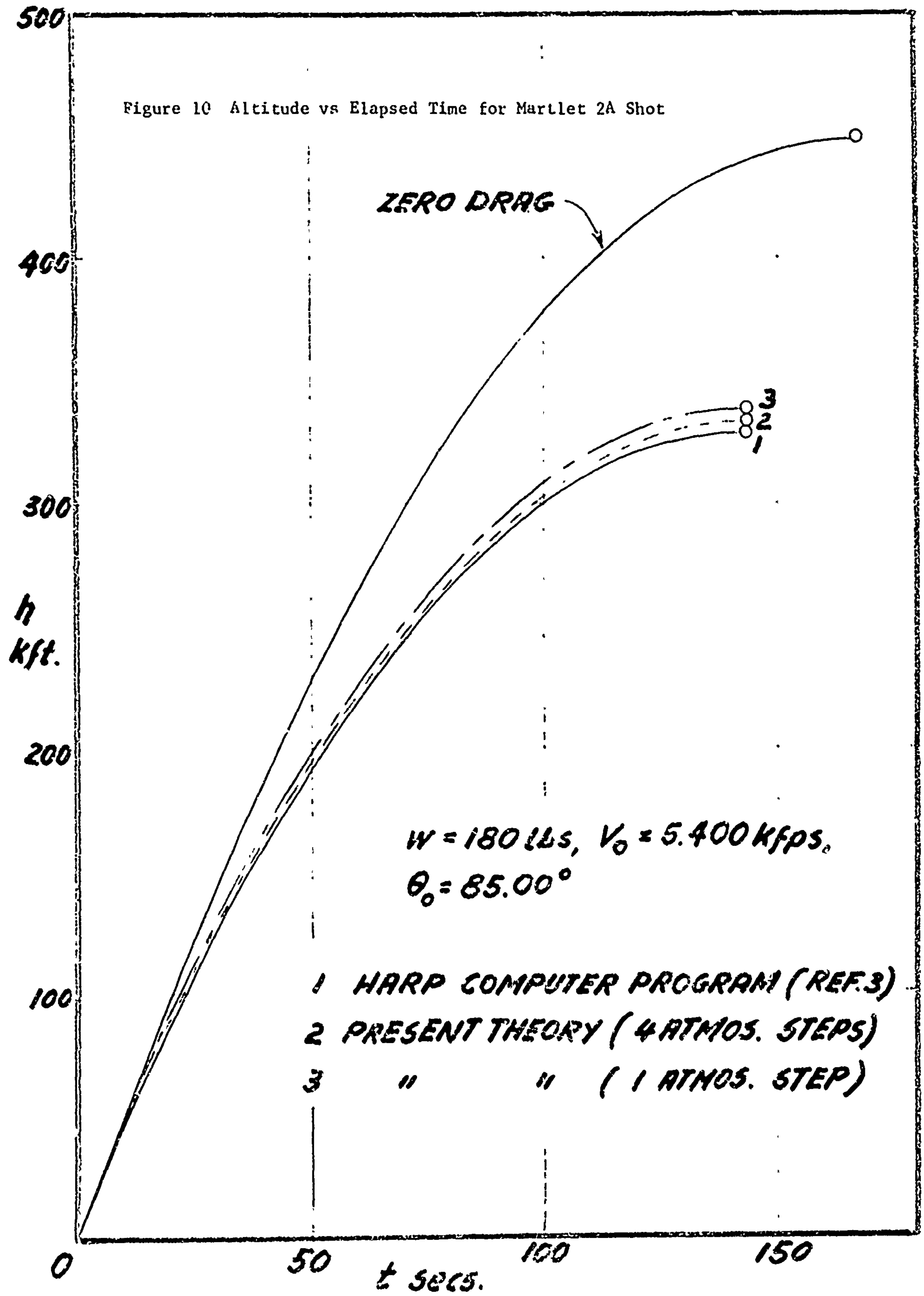


Figure 10 Altitude vs Elapsed Time for Martlet 2A Shot



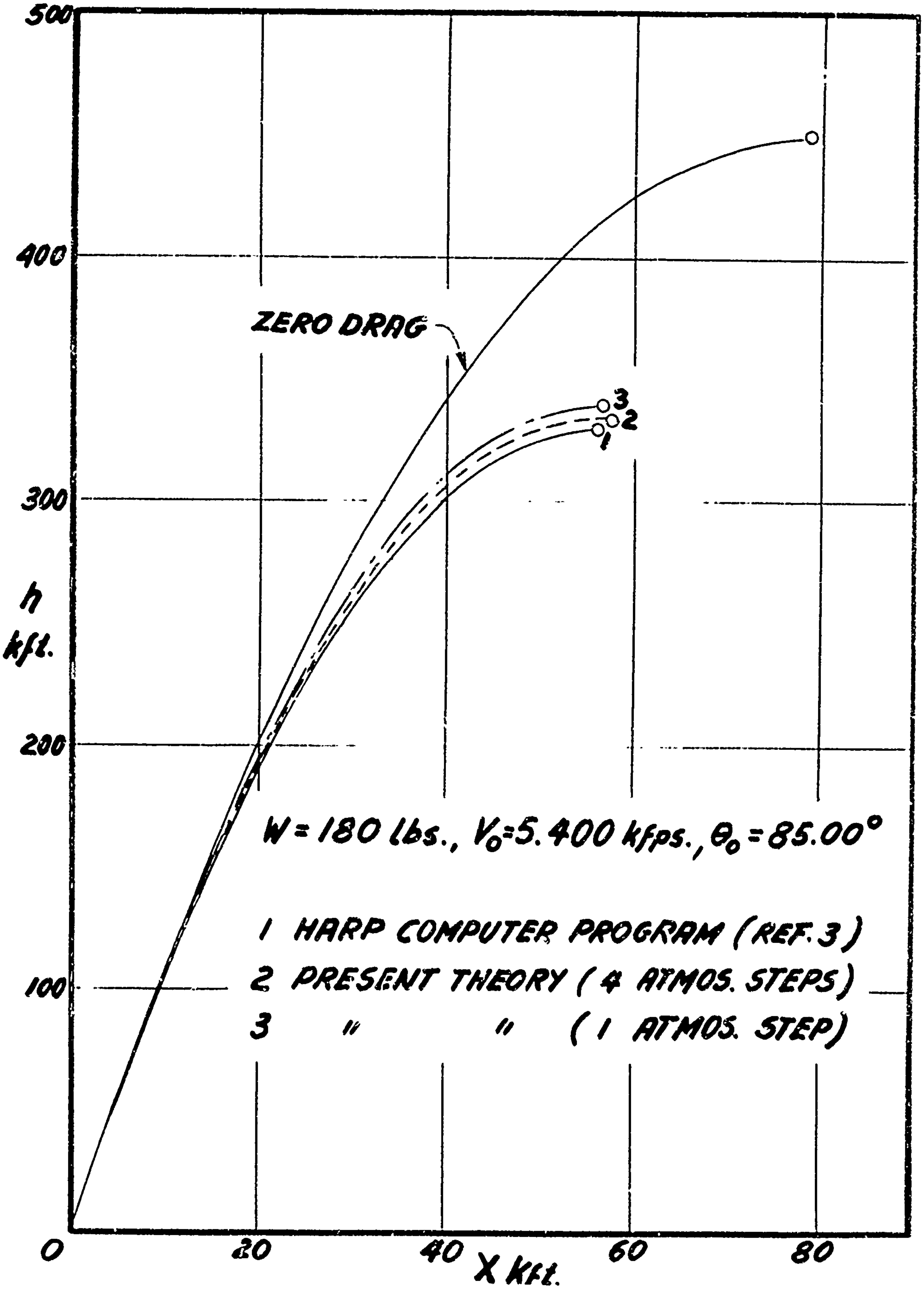


Figure 11 Martlet 2A Trajectories

A comparison that makes greater demands on the method is shown in Figure 12. Here a projectile is analyzed from launch to impact, with the entire trajectory low enough that measurable aerodynamic drag is experienced throughout. The present method with four atmospheric altitude increments for both climb to and descent from 80,000 ft and altitudes above 80,000 ft assumed non-atmospheric, is compared with the standard HARP computer program, and elapsed time, velocity, and altitude are plotted against range. Calculations are given in Appendix C.

3.5 DISCUSSION

The calculation procedure is very simple and, even for a choice of several altitude increments in the atmosphere, it can be carried out quickly using just a slide rule or a slide rule and trigonometric tables. Despite this simplicity, the curves of Figures 8, 9, 10, 11 and 12 show that it produces accurate results, agreeing closely both with actual measurements and with more exact numerical procedures.

Figure 8 shows that a four-step atmospheric calculation predicts the observed trajectory of Martlet 2A IOWA very closely, with the apogee only 1.8% higher than that observed.

Figures 9, 10 and 11 show that the four-step atmospheric calculation for a 180 lb Martlet 2A gives results nearly identical with those of the HARP computer program. Moreover, even the one-step atmospheric calculation gives good accuracy in this example, the velocity values and elapsed times lying quite close to the HARP curves, and the trajectory having an apogee only 3.0% high.

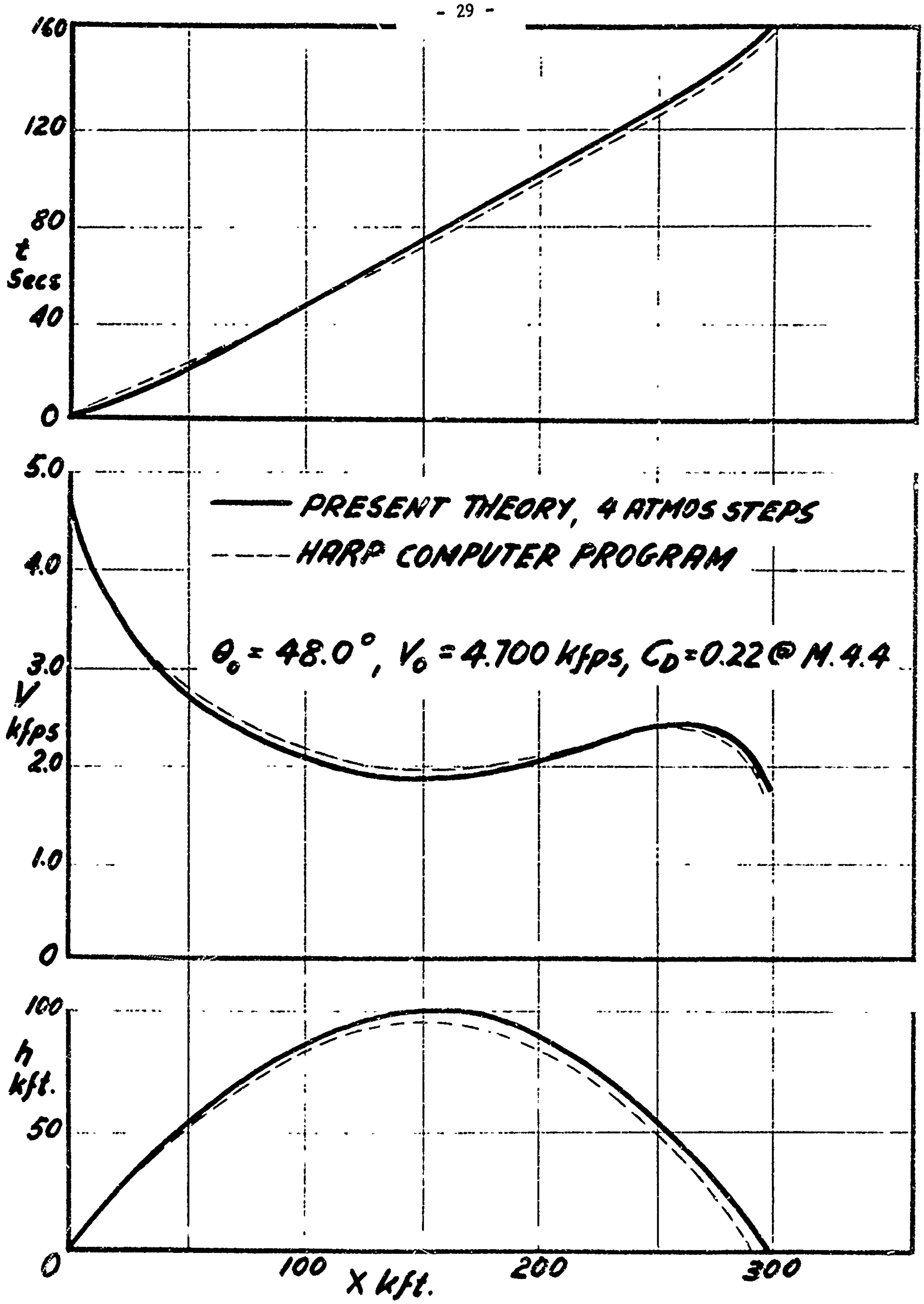


Figure 12 Trajectory Comparisons for Projectile

Figure 12 shows that even a low-altitude projectile trajectory is calculated from launch to impact with good accuracy by the method. Errors in calculated quantities at points along the trajectory are generally less than 5%, and in some of the more important quantities the errors are: range (+1.9%), apogee (+4.2%), range at apogee (+1.1%), velocity at apogee (-3.5%), time to impact (+4.1%), time to apogee (+3.3%), velocity at impact (-9.0%). The larger error in impact velocity is caused by matching the formula for drag coefficient, $C_D = \frac{K}{M}$, to the curve used in the computer program at M = 4.4. This gives excellent agreement at the higher Mach numbers, but produces higher drag at the lower Mach numbers preceding impact.

4.0 ROCKET POWERED TRAJECTORIES

The altitude change during the burning of a rocket motor stage of a vehicle is in general small enough to permit neglect of the change in g during motor burning. For altitudes below about 160 kft, however, effects of aerodynamic drag cannot be neglected, and the vehicle velocity increment ΔV_n resulting from rocket thrust is then given by Eq. (2.3), with the drag velocity decrement ΔV_{D_n} given by Eq. (2.11) and the gravity velocity decrement ΔV_{g_n} given by Eqs. (3.9) and (3.10). In this section the evaluation of ΔV_n from Eq. (2.3) and the determination of other rocket-powered trajectory parameters are described.

4.1 EVALUATION OF ELEVATION ANGLE

Before ΔV_{D_n} can be evaluated from Eq. (2.11), a suitable determination of the mean value $\overline{\sin \theta}_n$ must be made for a rocket-powered trajectory, and the dependence of mass m on altitude h during thrusting must be expressed so that the integral can be evaluated.

As in the glide trajectories of the previous section, the determination of $\overline{\sin \theta}_n$ is approached by neglecting the direct effect of drag on elevation angle. It is shown in Appendix D that in zero-drag, constant-g conditions, the differential equation governing the variation of elevation angle θ with time t under the action of thrust F is

$$\frac{d^2\theta}{dt^2} + (2 \tan \theta - \alpha \sec \theta)\left(\frac{d\theta}{dt}\right)^2 = 0 \qquad (4.1)$$

where

$$\alpha = \frac{F}{mg}$$

For the constant thrust rocket motors under consideration here, α varies with time and Eq. (4.1) is a fairly complicated nonlinear differential equation. Two special cases of the equation can, however, be treated easily. If $\alpha = 0$, of course, Eq. (4.1) governs zero-drag glide trajectories and the solution of Eqs. (3.5) and (3.6) is obtained. If α = constant, it is shown in Appendix D that Eq. (4.1) is easily reduced to the integral form:

$$\int_{\theta_{n-1}}^{\theta_n} \frac{\cos^{\alpha-2} \theta \, d\theta}{(1 + \sin \theta)^{\alpha}} = - \frac{\cos^{\alpha-1} \theta_{n-1} \Delta \tau_n}{(1 + \sin \theta_{n-1})^{\alpha}} \tag{4.2}$$

where

$$\Delta \tau_n = \frac{g \Delta \tau_n}{V_{n-1}}$$

This is integrable by quadratures for $\alpha = 0, 1, 2, 3$, and it is readily evaluated graphically or numerically for other values of α. Eq. (4.2) shows that $\theta_n = \theta_n (\theta_{n-1}, \alpha, \Delta \tau_n)$. Now, Δh_n, defined by Eq. (3.7) is also given by the functional form $\Delta h_n = \Delta h_n (\theta_{n-1}, \alpha, \Delta \tau_n)$, so that θ_n can be expressed

$$\theta_n = \theta_n (\theta_{n-1}, \alpha, \Delta h_n) \tag{4.3}$$

Eq. (4.3) must reduce to the value given by Eqs. (3.5) or (3.6) for $\alpha = 0$, and must reduce to $\theta_n = \theta_{n-1}$ for $\alpha \to \infty$. It is therefore plausible to try as an approximation suitable for easy calculation the analytic forms

$$\sin \theta_n = \sqrt{\frac{\sin^2 \theta_{n-1} - f(\alpha) \Delta h_n}{1 - f(\alpha) \Delta h_n}} \tag{4.4}$$

or $$\cos\theta_n = \frac{\cos\theta_{n-1}}{\sqrt{1 - f(\alpha)\Delta h_n}} \tag{4.5}$$

where $$f(o) = 1, \quad f(\infty) = 0.$$

An exponential form for $f(\alpha)$ suggests itself, and it was determined empirically as follows. It is shown in Appendix D that, for constant α,

$$\Delta h_n = 2(\alpha + \sin\theta_{n-1}) \int_0^{\Delta\tau_n} \frac{\sin\theta \, d\tau}{\alpha + \sin\theta} + 2(\alpha^2 - 1) \int_0^{\Delta\tau_n} \frac{\Delta\tau \sin\theta \, d\tau}{\alpha + \sin\theta} \tag{4.6}$$

Eqs. (4.2) and (4.6) were integrated numerically to obtain exact solutions for constant α with which values obtained from Eq. (4.4) with different choices of $f(\alpha)$ could be compared. The final choice of $f(\alpha)$ was

$$f(\alpha) = e^{-0.250\alpha^{0.7}} \tag{4.7}$$

In Fig. 13 this function is compared with the exact solutions at $\Delta\tau_n$ = .25 for $\theta_{n-1} = 30^\circ$ and 60° over the relevant range of α. $f(\alpha)$ was chosen to lie somewhat below the exact curves for two reasons. The first is to take some account of the effect of drag, otherwise neglected in this analysis. The second is to relate to the actual constant-thrust rocket motors under consideration, for which α increases steadily from ignition to burn-out. In applying the analysis to such motors, a mean value of α,

$$\overline{\alpha}_n = \tfrac{1}{2}(\alpha_{n-1} + \alpha_n) \tag{4.8}$$

is used for simplicity, but this tends to make $f(\overline{\alpha}_n)$ too small, and therefore $\sin\theta_n$ too large. With $\sin\theta_n$ given by Eq. (4.4) and (4.7) $\overline{\sin\theta_n}$ is given by Eq. (3.13).

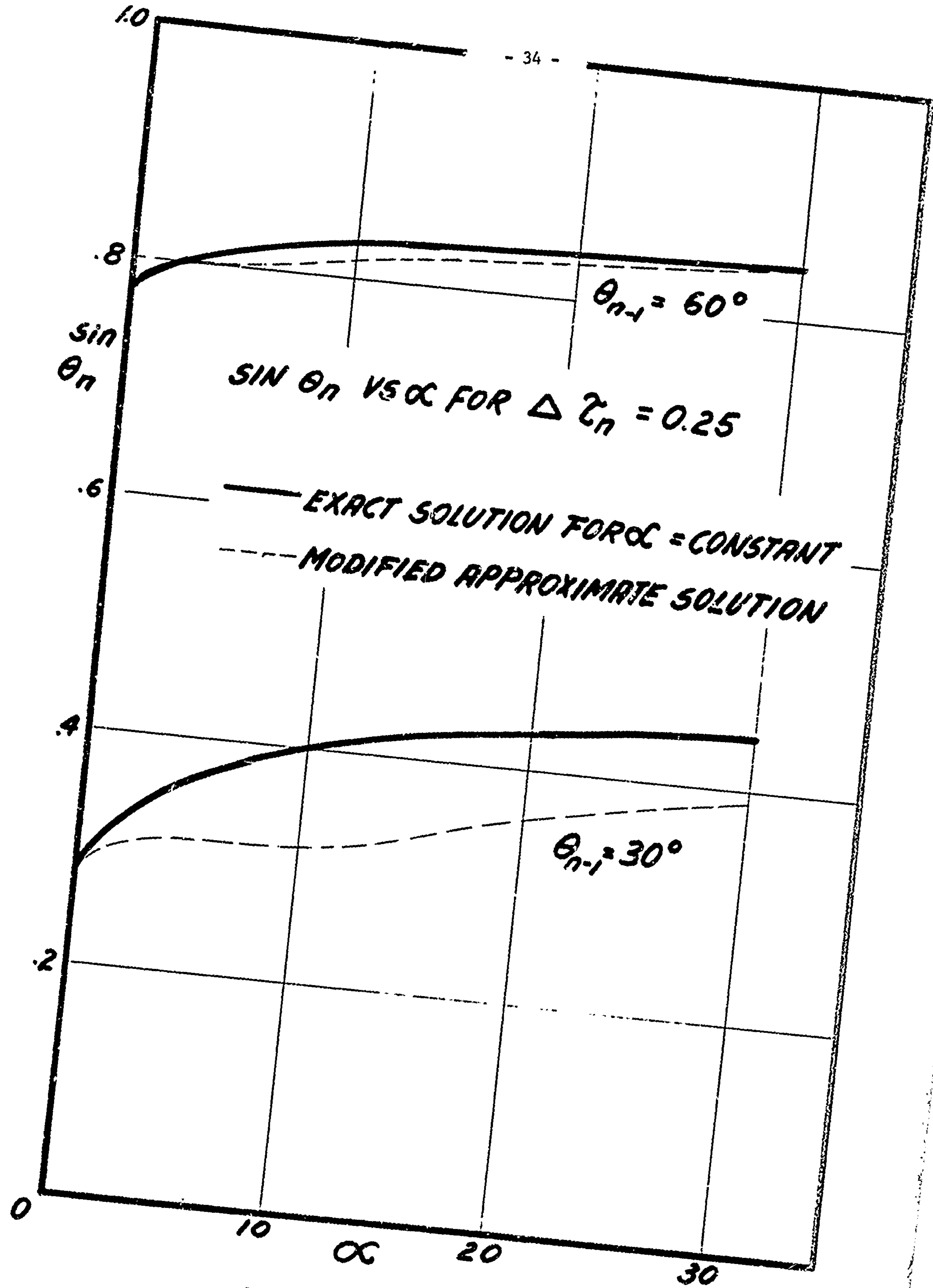


Figure 13 Sin θ_n vs α for $\Delta\tau_n = 0.25$

4.2 EVALUATION OF DRAG INTEGRAL

Since the drag velocity decrement is a small fraction of the velocity achieved under rocket thrust, the mass-altitude relation needed to determine it can be based with sufficient accuracy on the equivalent zero-drag trajectory, with $\sin\theta = \overline{\sin\theta_n}$. Eq. (2.1) can be written

$$m\frac{dV}{dt} = -V_{j_n}\frac{dm}{dt} - mg\,\overline{\sin\theta_n} \tag{4.9}$$

On multiplication by $\frac{dt}{m}$ and integration this gives

$$V = \frac{1}{\overline{\sin\theta_n}}\cdot\frac{dh}{dt} = V_{n-1} - V_{j_n}\ln\frac{m}{m_{n-1}} - g\,\overline{\sin\theta_n}\,(t - t_{n-1}) \tag{4.10}$$

A second integration, using the motor burning relation

$$m = m_{n-1} - b_n\,(t - t_{n-1}) \tag{4.11}$$

gives

$$h = h_{n-1} + \frac{(V_{n-1} + V_{j_n})\,\overline{\sin\theta_n}}{b_n}(m_{n-1} - m) + \frac{V_{j_n}\,\overline{\sin\theta_n}}{b_n}\,m\ln\frac{m}{m_{n-1}} - \frac{g\,\overline{\sin\theta_n}^{\,2}}{2b_n^{\,2}}(m_{n-1} - m)^2 \tag{4.12}$$

In Eq. (4.12) it should be noted that the last term is very small in comparison with the second term on the right side. Thus, their ratio is

$$\frac{g\,\overline{\sin\theta_n}\,(m_{n-1} - m)}{2b_n\,(V_{n-1} + V_{j_n})} = \frac{g\,\overline{\sin\theta_n}\,(t - t_{n-1})}{2(V_{n-1} + V_{j_n})} = \text{Order } (10)^{-2}$$

using typical values for the parameters. Accordingly, the last term can be neglected in the expression for m for use in Eq. (2.11). The remaining terms

are made dimensionless by multiplication by

$$\frac{b_n}{m_{n-1}\, V_{j_n} \sqrt{\sin \theta_n}}$$

giving:

$$\Delta H = \frac{b_n}{m_{n-1}\, V_{j_n} \sqrt{\sin \theta_n}} \,(h - h_{n-1}) = \left(\frac{1}{\upsilon_{j_n}} + 1\right)(1 - \mathfrak{m}) + \mathfrak{m} \ln \mathfrak{m} \tag{4.13}$$

where $$\upsilon_{j_n} = \frac{V_{j_n}}{V_{n-1}}\,, \quad \mathfrak{m} = \frac{m}{m_{n-1}}$$

Eq. (4.13) is plotted for relevant values of υ_{j_n} and $\mathfrak{m}$ in Fig. 14. (The subscript n of course refers to the evaluation of the function for an altitude increment $\Delta h_n = h_n - h_{n-1}$.)

Although a simple expression, Eq. (4.13) is not directly useful in evaluating Eq. (2.11), since it is transcendental and $\mathfrak{m}$ cannot be given explicitly as a function of ΔH. However, $\mathfrak{m}$ is unity for $\Delta H = 0$ and $\mathfrak{m}$ approaches zero as ΔH becomes large, so approximation by a function of the form

$$\mathfrak{m} = e^{-f(\upsilon_{j_n})\, \Delta H} \tag{4.14}$$

is plausible. It was found empirically by comparison with the curves of Fig. 14 that the function

$$f(\upsilon_{j_n}) = 1.10\, \upsilon_{j_n}^{\;0.60} \tag{4.15}$$

gives satisfactory agreement with Eq. (4.13). Thus, Eq. (4.14), using

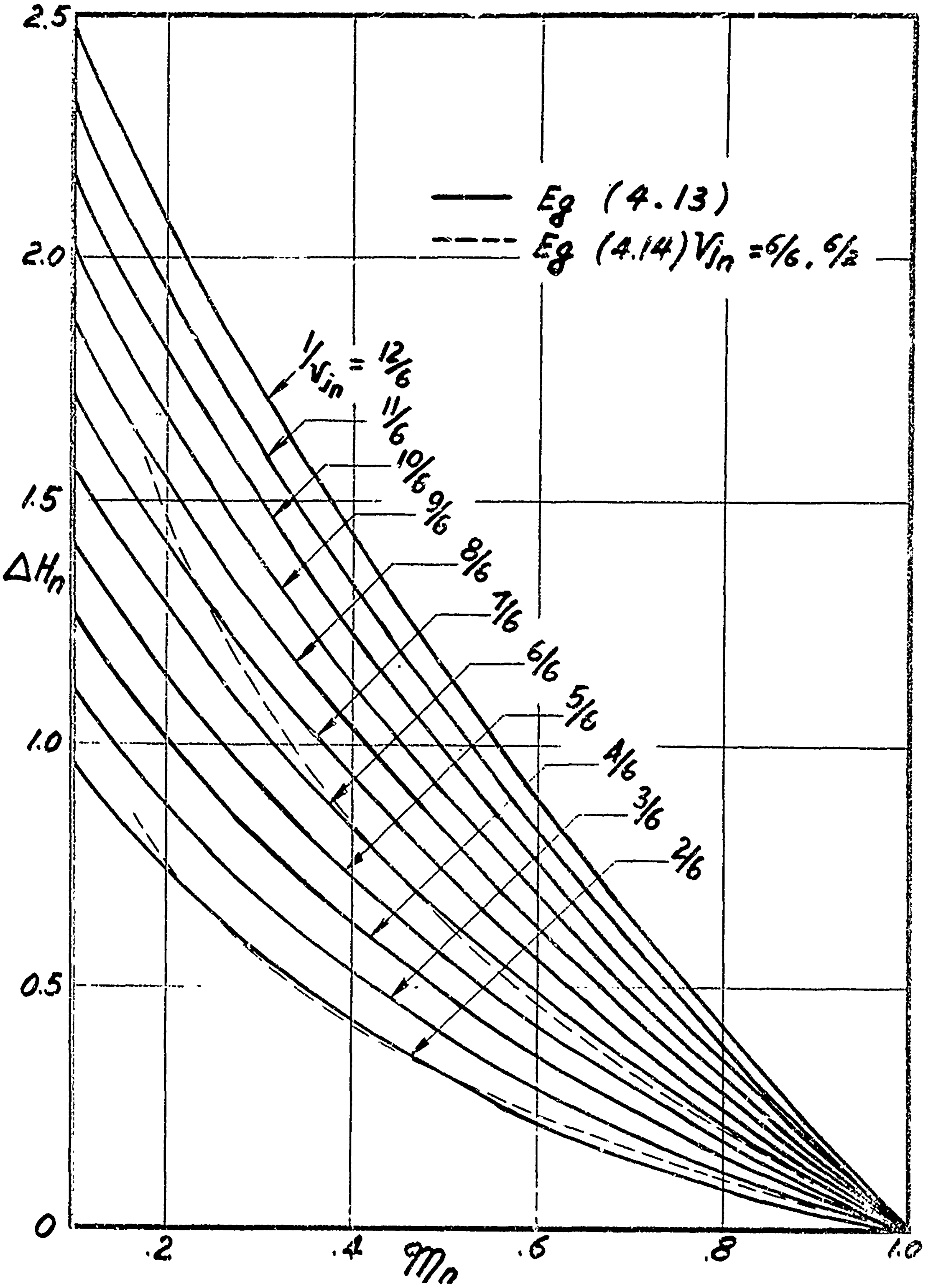


Figure 14 Altitude-Mass Relation for Zero-Drag, Constant-Thrust Motor

Eq. (4.15), is plotted on Fig. 14 for v_{j_n} = 1.0 and 3.0, and the agreement with Eq. (4.13) is seen to be good for η_ν greater than 0.3, the relevant range. Accordingly, Eq. (2.11) is put in the form

$$\Delta V_{D_n} = \frac{K \gamma A}{2m_{n-1} \overline{\sin \theta_n}} \Delta J_n \qquad (4.16)$$

where

$$\Delta J_n = \int_{h_{n-1}}^{h_n} \left(\frac{p}{a}\right) \frac{dh}{\eta_\nu} \qquad (4.17)$$

and η_ν is given by Eq. (4.14) and (4.15).

Unfortunately, however, the function for (p/a) given by Eq. (2.18) is not integrable by quadratures in combination with Eq. (4.14). It is therefore replaced for this integration by the function

$$\left(\frac{p}{a}\right) = \left(\frac{p}{a}\right)_{n-1} e^{-c(h - h_{n-1})} \qquad (4.18)$$

where c is determined empirically to give good agreement with the true variation of (p/a) over a selected altitude range. Eq. (4.18) would be exact for an isothermal atmosphere, and is a good approximation for an actual atmosphere over limited ranges of altitude. Thus, in Fig. 15, Eq. (4.18) is seen to give excellent agreement with the Cape Kennedy Standard Atmosphere for altitudes between 40 kft and 120 kft, with c = 0.04808/kft.

With the combination of Eq. (4.14) and (4.18), ΔJ_n can be evaluated:

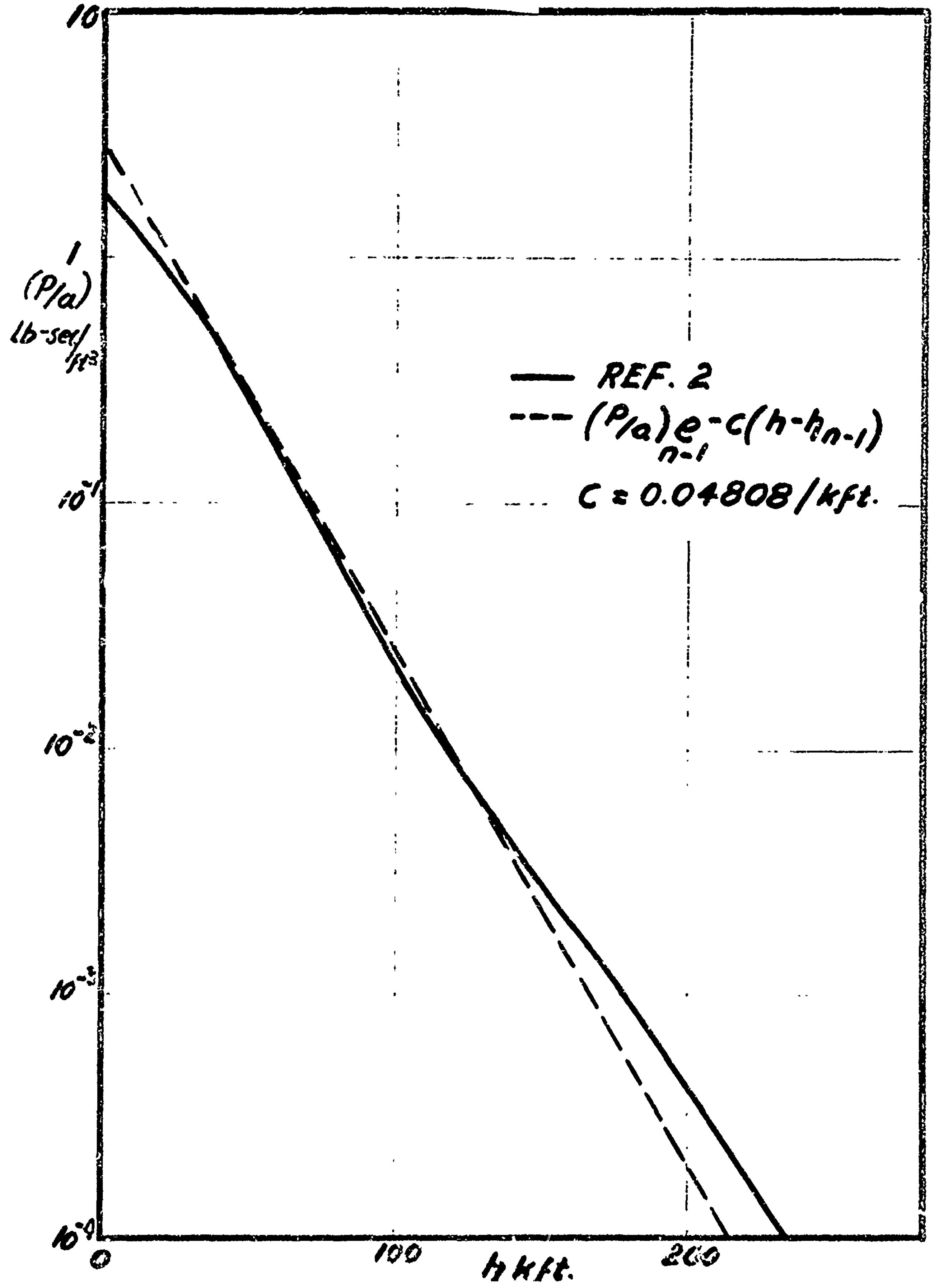


Figure 15 ($\frac{p}{a}$) vs h for Cape Kennedy Standard Atmosphere

$$\Delta J_n = \left(\frac{p}{a}\right)_{n-1} \int_{h_{n-1}}^{h_n} e^{\left\{-c + \frac{1.10}{\upsilon_{j_n}^{0.40}} \frac{b_n}{m_{n-1} V_{n-1} \overline{\sin\theta_n}}\right\}(h - h_{n-1})} dh$$

$$= \left(\frac{p}{a}\right)_{n-1} \frac{1}{\zeta_n} \left\{1 - e^{-\zeta_n \Delta h_n}\right\} \tag{4.19}$$

where

$$\zeta_n = c - \frac{1.10\ b_n}{\upsilon_{j_n}^{0.40} \cdot m_{n-1} V_{n-1} \overline{\sin\theta_n}} \tag{4.20}$$

4.3 DETERMINATION OF VELOCITY AND TRAJECTORY PARAMETERS

It is now possible to consider the evaluation of the net velocity increment ΔV_n arising from the effects of rocket motor thrust, aerodynamic drag, and gravity:

$$\Delta V_n = \Delta V_{F_n} - \Delta V_{D_n} - \Delta V_{g_n} \tag{2.3}$$

or

$$\Delta V_n = \Delta V_{F_n} - \Delta V_{D_n} - \frac{g \Delta h_n}{V_{n-1} + \frac{1}{2} \Delta V_n}$$

using Eq. (3.9) and (3.10). This is made non-dimensional by division by V_{n-1}:

$$\Delta\upsilon_n = \Delta\upsilon_{F_n} - \frac{\Delta \hbar_n}{2 + \Delta\upsilon_n} - \Delta\upsilon_{D_n} \tag{4.21}$$

where

$$\Delta\upsilon_{F_n} = -\upsilon_{j_n} \ln \eta_n,\ \Delta\upsilon_{D_n} = \frac{K \gamma A}{2m_{n-1} V_{n-1} \overline{\sin\theta_n}} \Delta J_n \tag{4.22}$$

Solving Eq. (4.21) for Δv_n:

$$\Delta v_n = -1 + \tfrac{1}{2}(\Delta v_{F_n} - \Delta v_{D_n}) + \sqrt{\left\{1 + \tfrac{1}{2}(\Delta v_{F_n} - \Delta v_{D_n})\right\}^2 - \Delta h_n} \qquad (4.23)$$

Eq. (4.23) reduces to Eq. (3.11) if $\Delta v_{F_n} = 0$ (no rocket thrust). The procedure for evaluating Eq. (4.23) is, however, different from the previous one. In evaluating Eq. (3.11), Δh_n was selected and therefore Δh_n was given, and the time increment Δt_n was part of the solution. In the present case, in which the burning of one rocket motor stage constitutes a trajectory increment, the time increment Δt_n is known, as is the mass decrement Δm_n, so Δh_n must be found from the mass-altitude relation.

In Eq. (4.12) there is no need to neglect the small final term at this stage of the calculations, since only simple algebra is involved. Therefore, on multiplication by $\frac{2g}{V^2_{n-1}}$, Eq. (4.12) becomes:

$$\Delta h_n = 2 v_{j_n} \, \delta_n \, \Delta H_n - (1 - m_n)^2 \, \delta_n^{\;2} \qquad (4.24)$$

where

$$\delta_n = \frac{g m_{n-1} \overline{\sin \theta_n}}{V_{n-1} \, b_n}$$

and ΔH_n is given by Eq. (4.13) or Fig. 14. The calculation procedure can now be described. It is convenient to introduce a parameter ε_{n-1} defined by:

$$\varepsilon_{n-1} = \frac{V_{n-1} \, b_n}{g \, m_{n-1}} \qquad (4.25)$$

In terms of ε_{n-1},

$$\overline{\alpha}_n = \tfrac{1}{2}(\alpha_{n-1} + \alpha_n) = \frac{V_{j_n} b_n}{2g}\left(\frac{1}{m_{n-1}} + \frac{1}{m_n}\right) = \frac{\mathcal{V}_{j_n}\varepsilon_{n-1}}{2}\left(1 + \frac{1}{\mathcal{m}_n}\right) \quad (4.26)$$

and

$$\delta_n = \frac{\overline{\sin\theta_n}}{\varepsilon_{n-1}}$$

A complete rocket-powered stage is calculated as one trajectory increment. The known initial parameters are V_{n-1}, t_{n-1}, h_{n-1}, θ_{n-1}, X_{n-1}, g, K, γ, A, m_{n-1}, b_n, c, V_{jn}, $(p/a)_{n-1}$. Δt_n is known, and this determines t_n and Δm_n. $\mathcal{V}_{j_n}$ and $\mathcal{m}_n$ are calculated and ΔH_n is found from Fig. 14. ε_{n-1} and $\overline{\alpha}_n$ are calculated, and a trial value of $\overline{\sin\theta_n}$ is estimated, based on the knowledge of sin θ_{n-1} and $\overline{\alpha}_n$. Using this, δ_n is calculated from Eq. (4.27) and $\Delta\mathcal{h}_n$ from Eq. (4.24). $f(\overline{\alpha}_n)$ is then found from Eq. (4.7) and cos θ_n is calculated from Eq. (4.5). This determines sin θ_n and leads to a new value of $\overline{\sin\theta_n}$. If this differs from the estimated value, the process is repeated from that point, using the new value of $\overline{\sin\theta_n}$. Only the one iteration should ever be required. Next Δh_n is calculated from $\Delta\mathcal{h}_n$, giving h_n. $\mathcal{J}_n$ is calculated from Eq. (4.20) and used to calculate ΔJ_n from Eq. (4.19). This gives $\Delta\mathcal{V}_{D_n}$ from Eq. (4.22), and $\Delta\mathcal{V}_{F_n}$ is then calculated, and $\Delta\mathcal{V}_n$ is determined from Eq. (4.23). This gives ΔV_n and V_n. Range increment ΔX_n can now be calculated from Eq. (3.14) and this gives range X_n, completing the stage calculation.

As an example, the method was applied to a Martlet 4, a gun-launched three-stage rocket, from launch to burn out of the second stage (the remainder of the trajectory is high enough to require consideration of

the g-variation). It was launched at 6.000 kfps at an elevation angle of 33.0°. The first stage motor was ignited at 40.0 kft, and the second stage motor immediately after burn-out of the first stage. Three altitude increments were used in calculating the glide trajectory before ignition. In Table 2 the results of the calculations are compared with the output of the standard HARP Computer Program for the same example obtained from Ref. 4. Vehicle parameters and trajectory calculations, and the output of the Computer Program, are given in Appendix E.

TABLE 2

Martlet 4 Trajectory Calculations for HARP Case 1046

		V_n kfps	X_n kft	h_n kft	t_n sec	θ_n °
Launch		6.000	0	0	0	33.0
Stage 1 Ignition	HARP	4.822	67.2	40.0	14.61	28.92
	Present	4.808	66.4	40.0	14.67	28.8
	Error	-.29%	-1.34%	-	+.41%	-.42%
Stage 2 Ignition	HARP	12.155	174.8	93.9	29.61	26.02
	Present	12.21	172.8	97.1	29.67	27.7
	Error	+.45%	-1.14%	+3.41%	+.20%	+6.46%
Stage 2 Burn-out	HARP	18.621	311.8	158.7	39.61	25.35
	Present	18.84	306.8	166.9	39.67	27.3
	Error	+1.18%	-1.67%	+5.16%	+.15%	+7.69%

4.4 DISCUSSION

Table 2 and Appendix E show that rocket-powered trajectories through the atmosphere can be accurately and easily calculated by slide rule by the present method. In the example the velocity and range at burn-out of Stage 2 were given very accurately, the altitude and elevation angle less accurately, both being too high. The time was of course given accurately, since it was known except for the initial glide trajectory.

The high values of h_n and θ_n, although acceptable, lead to over-estimates of apogee, or of orbital height after Stage 3 firing, and this suggests further work in improving the method of determining $\overline{\sin \theta_n}$, so as to decrease its value towards the correct one while retaining a simple dependence on the relevant parameters.

5.0 HIGH ALTITUDE AND ORBITAL TRAJECTORIES

Rocket-powered vehicles will in general reach very high altitudes or go into earth orbits. The upper parts of their trajectories must then be calculated by recognizing that gravitational force is a central force varying inversely as the square of the central distance. No aerodynamic drag need be considered unless such problems as the gradual decay of orbits are being studied, and any high altitude use of rocket thrust can be treated as a constant-g problem using an appropriate value of g, because of the short duration of the firing.

The problem then becomes that of determining zero-drag glide trajectories under the action of an inverse-square central force. The basic equations governing this motion are simple and lead to explicit solutions for trajectory parameters, so there is no need to seek even simpler approximate solutions. In this section the usual solutions are merely derived to complete the set of trajectory equations. No examples are worked, since the solutions are not new.

5.1 EFFECT OF EARTH ROTATION

In previous sections of the report, trajectory analysis was given in terms of vehicle velocity V relative to earth. This was suitable over short distances for which gravity could be considered a constant vector. When the true central force nature of gravity is considered, the trajectory analysis must be based on the vehicle absolute velocity of magnitude U, or in vector form:

$$\vec{U} = \vec{V} + \vec{U}_E \tag{5.1}$$

where $\vec{U_E}$ is the vector velocity of the earth's surface at the gun muzzle. Since $\vec{U_E}$ and $\vec{V}$ are in different directions, it is convenient to express relations between components. These relations, in the context of this report, would be applied at the point on the vehicle trajectory where the constant-ε analysis of previous sections is replaced by the analysis of this section. The relations can be obtained from Fig. 16. First, using the cosine law:

$$U^2 = V^2 + U_E{}^2 + 2VU_E \cos \chi \tag{5.2}$$

But, from the right triangles of the diagram it can be seen that

$$V \cos \theta \cos \psi_v = V \cos \chi \tag{5.3}$$

so that, cancelling V and substituting for $\cos \chi$ in Eq. (5.2):

$$U^2 = V^2 + U_E^2 + 2VU_E \cos \theta \cos \psi_v \tag{5.4}$$

where ψ_v is the azimuth angle of the gun. Eq. (5.4) gives U. The absolute path angle σ to the local horizontal is given by

$$U \sin \sigma = V \sin \theta \tag{5.5}$$

The absolute azimuth angle ψ_u is given by

$$U \cos \sigma \cos \psi_u = V \cos \theta \cos \psi_v + U_E \tag{5.6}$$

5.2 FUNDAMENTAL EQUATIONS AND SOLUTIONS

Because only the central force of gravity acts on the vehicle its trajectory remains in a plane passing through the earth's centre, and Fig. 17 defines the variables of the trajectory. The equations of motion of the vehicle for the r- and Ø- directions are written, for unit mass:

<u>r-direction</u> $$\frac{d^2r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = -g_E\left(\frac{r_E}{r}\right)^2 \qquad (5.7)$$

<u>Ø-direction</u> $$r\frac{d^2\phi}{dt^2} + 2\frac{dr}{dt}\frac{d\phi}{dt} = 0 \qquad (5.8)$$

The second terms on the left side of the two equations are the centripetal and coriolis accelerations, respectively, and g_E is the value of g at the earth's surface, used in the previous sections of the report.

If Eq. (5.8) is multiplied by r, it can be written:

$$\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = 0$$

or

$$r^2\frac{d\phi}{dt} = C\text{, constant} = rU\cos\sigma \qquad (5.9)$$

where $$C = r_{n-1}\,U_{n-1}\cos\sigma_{n-1} \qquad (5.10)$$

$\frac{d\phi}{dt}$ can be eliminated from Eq. (5.7) using Eq. (5.9):

$$\frac{d^2r}{dt^2} - \frac{C^2}{r^3} = -g_E\left(\frac{r_E}{r}\right)^2 \qquad (5.11)$$

Now, $\frac{dr}{dt} = U\sin\sigma = U_r$. Therefore

$$\frac{d^2r}{dt^2} = \frac{dU_r}{dt} = U_r\frac{dU_r}{dr} = \frac{C^3}{r^3} - \frac{g_E\,r_E^2}{r^2} \qquad (5.12)$$

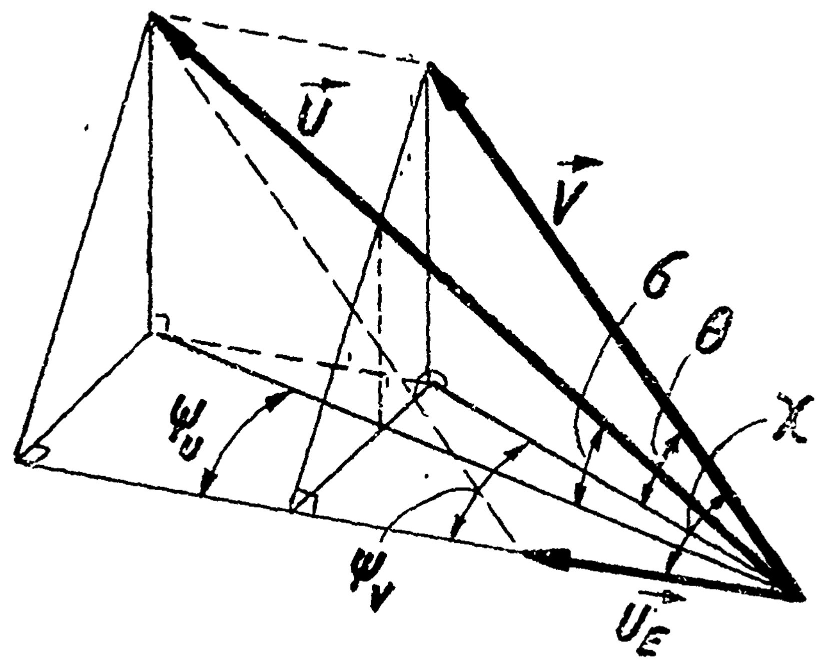


Figure 16 Vehicle Velocity Diagram

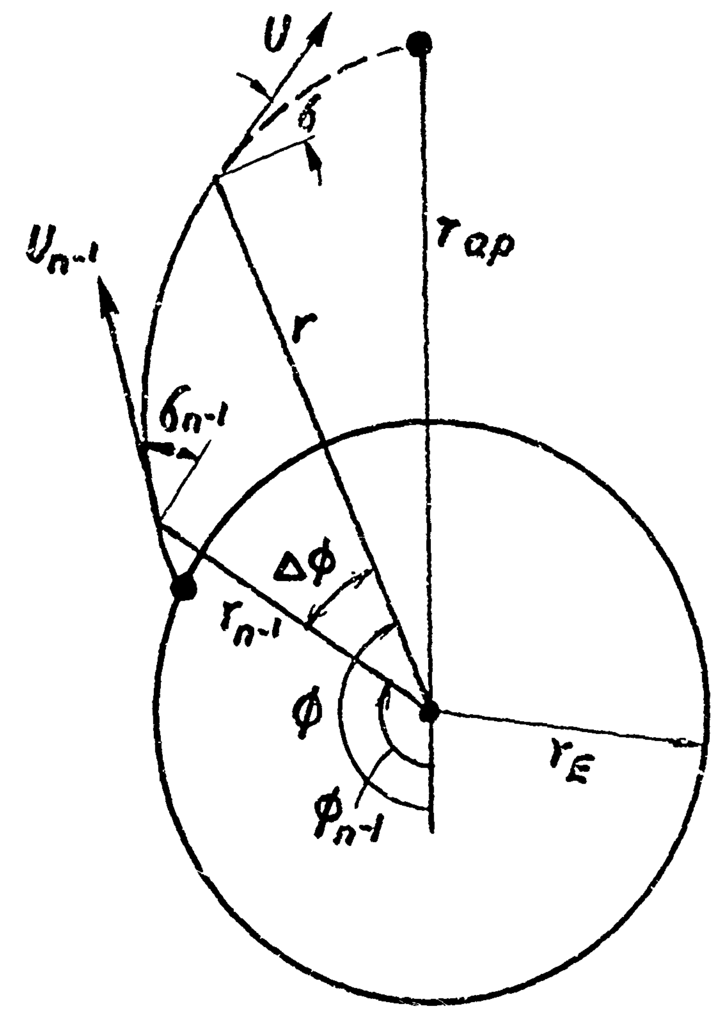


Figure 17 Trajectory Variables

Eq. (5.12) can be integrated directly to give

$$U_r^{\,2} - U_{r_{n-1}}^{\,2} = \left(\frac{1}{r_{n-1}} - \frac{1}{r} \right) \left\{ C^2 \left(\frac{1}{r_{n-1}} + \frac{1}{r} \right) - 2g_E\, r_E^{\,2} \right\} \tag{5.13}$$

Since the altitude change of the vehicle is of primary interest, it is convenient to introduce

$$\Delta\hbar = \frac{\Delta h}{r} = 1 - \frac{r_{n-1}}{r} \tag{5.14}$$

In terms of $\Delta\hbar$, Eq. (5.13) appears in the form

$$U_r^{\,2} - U_{r_{n-1}}^{\,2} = -\Delta\hbar \left\{ \frac{2g_E r_E^{\,2}}{r_{n-1}} - U_{\phi_{n-1}}^2 \,(2 - \Delta\hbar) \right\} \tag{5.15}$$

where

$$U_\phi = U \cos \sigma$$

At apogee, $U_r = 0$, so that Eq. (5.15) can be solved for $\Delta\hbar_{ap}$:

$$\Delta\hbar_{ap} = \left\{ 1 - \left(\frac{g_E\, r_E^{\,2}}{r_{n-1} U_{\phi_{n-1}}^{\,2}} \right) \right\} + \sqrt{\left\{ 1 - \left(\frac{g_E\, r_E^{\,2}}{r_{n-1} U_{\phi_{n-1}}^{\,2}} \right) \right\}^2 + \tan^2 \sigma_{n-1}} \tag{5.16}$$

Eqs. (5.15) and (5.9) determine vehicle velocity as a function of altitude. It is necessary to relate altitude to elapsed time. This can be done using

$$dt = \frac{dr}{U_r} = \frac{r^2 d\,\Delta\hbar}{r_{n-1} U_r} = \frac{r_{n-1} d\,\Delta\hbar}{(1 - \Delta\hbar)^2 U_r} \tag{5.17}$$

Eq. (5.17) can be integrated directly, using Eq. (5.15), to give

$$\Delta t = \frac{r_{n-1}}{U_{\phi_{n-1}} S}\left[\tan\sigma_{n-1} - \frac{1}{(1-\Delta h)}\sqrt{\tan^2\sigma_{n-1} - 2\left\{\frac{g_E r_E^2}{r_{n-1}U_{\phi_{n-1}}^2} - 1\right\}\Delta h - \Delta h^2}\right]$$

$$+ \frac{g_E r_E^2}{U_{\phi_{n-1}}^3 S^{3/2}}\left[\sin^{-1}\left\{\frac{-2S + \frac{2g_E r_E^2}{r_{n-1}U_{\phi_{n-1}}} 2(1-\Delta h)}{-(1-\Delta h)Q}\right\} - \sin^{-1}\left\{\frac{-2S + \frac{2g_E r_E^2}{r_{n-1}U_{\phi_{n-1}}^2}}{-Q}\right\}\right] \quad (5.18)$$

where

$$S = \frac{2g_E r_E^2}{r_{n-1}U_{\phi_{n-1}}^2} - \sec^2\sigma_{n-1}$$

$$Q = \sqrt{4\tan^2\sigma_{n-1} + \left\{\frac{2g_E r_E^2}{r_{n-1}U_{\phi_{n-1}}^2} - 2\right\}^2} \quad (5.19)$$

In order to find the range of the vehicle, it is necessary to know the change in ϕ. This can be found using Eq. (5.9):

$$\frac{d\phi}{dt} = \frac{d\phi}{dr} U_r = \frac{C}{r^2}$$

Therefore

$$d\phi = \frac{Cdr}{r^2 U_r} = \frac{C\, d\Delta h}{r_{n-1} U_r} \quad (5.20)$$

Eq. (5.20) can be integrated directly, using Eq. (5.15), to give

$$\Delta\phi = \sin^{-1}\left\{\frac{2\Delta h + 2\left(\frac{g_E r_E^2}{r_{n-1}U_{\phi_{n-1}}^2} - 1\right)}{Q}\right\} - \sin^{-1}\left\{\frac{2}{Q}\left(\frac{g_E r_E^2}{r_{n-1}U_{\phi_{n-1}}^2} - 1\right)\right\}. \quad (5.21)$$

Referring to Fig. 18 the vehicle range increment ΔX is the great circle distance between the point 1 intercepted at time t at the earth's surface by radius r and the position 2 at time t of the point 3 on the earth's surface intercepted at time t_{n-1} by radius r_{n-1}:

$$\Delta X = r_e \Delta \phi' \tag{5.22}$$

$\Delta\phi'$ can be obtained by solving the spherical triangle 123. In this, $\Delta\phi$ is known from Eq. (5.21), ψ'_U is given by Eq. (5.6) and great circle arc 23 and angle 231 can be found, since the latitude of 3 and the small circle (eastward) distance 32 travelled by point 3 in the time Δt, given by Eq. (5.18), are known.

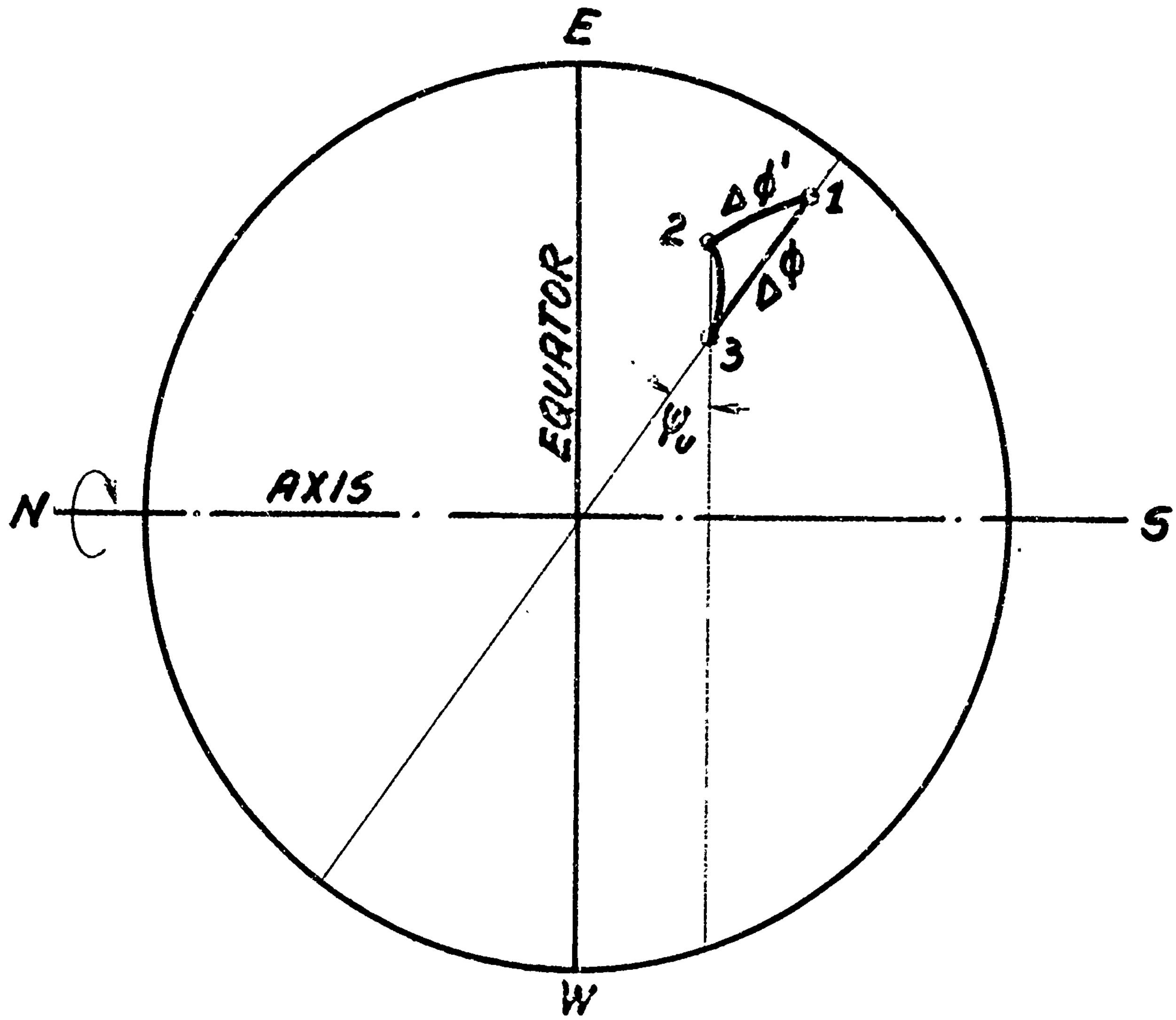


Figure 18 Spherical Geometry for Range

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APPENDIX A

Trajectory Equations for Zero Drag, Zero Thrust, Constant g.

Newton's 2nd Law for X-Direction (see Figure 1)

$$\frac{d}{dt}\left(m\frac{dx}{dt}\right) = m\frac{d^2X}{dt^2} = 0$$

Therefore

$$\frac{dX}{dt} = \text{constant} = V_{n-1}\cos\theta_{n-1} = V\cos\theta \qquad \text{(A-1)}$$

and

$$\Delta X_n = X_n - X_{n-1} = V_{n-1}\cos\theta_{n-1}(t_n - t_{n-1}) \qquad \text{(A-2)}$$

Newton's 2nd Law for h-Direction

$$-mg = \frac{d}{dt}\left(m\frac{dh}{dt}\right) = m\frac{d^2h}{dt^2} = \text{constant}$$

Therefore

$$\frac{dh}{dt} = V_{n-1}\sin\theta_{n-1} - g(t - t_{n-1}) = V\sin\theta \qquad \text{(A-3)}$$

and

$$\Delta h_n = h_n - h_{n-1} = V_{n-1}\sin\theta_{n-1}(t_n - t_{n-1}) - \frac{g}{2}(t_n - t_{n-1})^2 \qquad \text{(A-4)}$$

Introduce

$$\Delta\mathcal{X}_n = \frac{2g\Delta X_n}{V_{n-1}^2}, \quad \Delta\hbar_n = \frac{2g\Delta h_n}{V_{n-1}^2} \qquad \text{(A-5)}$$

and substitute for $(t_n - t_{n-1})$ from Eq. (A-2) in Eq. (A-4):

$$\Delta\hbar_n = \tan\theta_{n-1}\Delta\mathcal{X}_n - \tfrac{1}{4}\sec^2\theta_{n-1}\Delta\mathcal{X}_n^{\,2} \qquad \text{(3.4)}$$

on dividing Eq. (A-3) by Eq. (A-1) an expression for tan θ is obtained:

$$\tan\theta = \tan\theta_{n-1} - \frac{g(t - t_{n-1})}{V_{n-1}\cos\theta_{n-1}} \qquad \text{(A-6)}$$

If θ is set equal to θ_n and $(t_n - t_{n-1})$ is elimated using Eq. (A-2) the result can be written:

$$\tan\theta_n = \tan\theta_{n-1} - \tfrac{1}{2}\sec^2\theta_{n-1}\,\Delta\chi_n$$

or

$$\Delta\chi_n = \frac{2(\tan\theta_{n-1} - \tan\theta_n)}{\sec^2\theta_{n-1}} \tag{A-7}$$

Again, if Eq.(3.4) is solved for $\Delta\chi_n$, the result is:

$$\Delta\chi_n = \frac{2\left(\tan\theta_{n-1} - \sqrt{\tan^2\theta_{n-1} - \sec^2\theta_{n-1}\Delta h_n}\right)}{\sec^2\theta_{n-1}} \tag{A-8}$$

On comparing Eqs. (A-7) and (A-8) it is seen that

$$\tan\theta_n = \sqrt{\tan^2\theta_{n-1} - \sec^2\theta_{n-1}\,\Delta h_n} \tag{A-9}$$

With a little rearrangement, using trigonometric identities, this can be put in the form:

$$\sin\theta_n = \sqrt{\frac{\sin^2\theta_{n-1} - \Delta h_n}{1 - \Delta h_n}} = \sqrt{1 - \frac{\cos^2\theta_{n-1}}{1 - \Delta h_n}} \tag{3.5}$$

or

$$\cos\theta_n = \frac{\cos\theta_{n-1}}{\sqrt{1 - \Delta h_n}} \tag{3.6}$$

If Eq.(A-4) is multiplied through by $\dfrac{2g}{V_{n-1}^2\sin^2\theta_{n-1}}$ the result is

$$\left(\frac{2g\,\Delta h_n}{V_{n-1}^2\sin^2\theta_{n-1}}\right) = 2\left(\frac{g\Delta t_n}{V_{n-1}\sin\theta_{n-1}}\right) - \left(\frac{g\Delta t_n}{V_{n-1}\sin\theta_{n-1}}\right)^2 \tag{A-10}$$

or, if Eq.(A-2) is used to substitute for Δt_n:

$$\left(\frac{2g\ \Delta h_n}{V_{n-1}^2 \sin^2\theta_{n-1}}\right) = 2\left(\frac{g\ \Delta X_n}{V_{n-1}^2 \sin\theta_{n-1} \cos\theta_{n-1}}\right) - \left(\frac{g\ \Delta X_n}{V_{n-1}^2 \sin\theta_{n-1} \cos\theta_{n-1}}\right)^2 \qquad \text{(A-11)}$$

Both Eq.(A-10) and (A-11) have the form

$$\eta = 2\ \xi - \xi^2 \qquad (3.16)$$

giving a universal zero-drag, constant-g trajectory equation.

APPENDIX B

Calculations for Martlet 2A Shots

Given: $g = 32.15$ ft/sec^2, $\gamma = 1.400$, $A = \frac{\pi}{4}\left(\frac{5.00}{144}\right)^2 = 0.1364$ sq.ft., $\bar{K} = 0.920$

$V_o = 5.400$ kfps, $\theta_o = 85.00°$

Case 1: $W = 170.0$ lbs, $m = \frac{170.0}{32.15} = 5.29$ slugs, $\frac{K\gamma A}{2m} = \frac{0.920(1.400)0.1364}{2(5.29)} = 16.62\,(10)^{-3}$ ft^3/lb-sec^2

Choose: $h_1 = 10$ kft., $h_2 = 30$ kft., $h_3 = 60$ kft., $h_4 = 180$ kft.

Column	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Source	Given	Given	Col.14	Eq.3.7	Tables	Tables	Eq.3.6	Eq.3.13	Tables	Fig.5	Eq.3.8	Eq.3.12	Eq.3.11	Eq.3.12	Eq.3.14	Col.15
n	h_n	Δh_n	V_n	$\Delta \bar{k}_n$	θ_n	$\sin\theta_n$	$\cos\theta_n$	$\overline{\sin\theta}_n$	$\bar{\theta}_n$	ΔI_n	ΔV_{D_n}	$\frac{1}{2}\Delta \nu_{D_n}$	$\Delta \nu_n$	ΔV_n	ΔX_n	X_n
-	kft	kft	kfps	-	°	-	-	-	°	kip-sec/ft^2	kfps	-	-	kfps	kft	kft
0	0	-	5.400	-	85.00	.9962	.0872	-	-	-	-	-	-	-	-	0
1	10	10	5.068	.0220	84.94	.9961	.0881	.99615	84.97	16.2	.270	.0250	-.0514	-.332	.880	.880
2	30	20	4.618	.0500	84.81	.9959	.0904	.9960	84.88	19.1	.319	.0313	-.0887	-.450	1.793	2.673
3	60	30	4.216	.0903	84.56	.9955	.0948	.9957	84.68	11.0	.184	.0199	-.0869	-.402	2.792	5.465
4	180	120	3.107	.4337	82.76	.9920	.1260	.9937	83.57	3.4	.057	.0067	-.2630	-1.109	13.53	19.00

For $h > 180$ kft., Figure 7 is used. Here

$$\Delta h_5 = \frac{(V_4 \sin\theta_4)^2}{2g}\,\eta = \frac{(3.107\,[.9920])^2}{.0643}\,\eta = 147.8\,\eta$$

$$\Delta x_5 = \frac{V_4^2 \sin\theta_4 \cos\theta_4}{g}\,\xi = \frac{(3.107)^2 .9920\,(.1260)}{.03215}\,\xi = 37.6\,\xi$$

η	Δh_5 kft	h_5 kft	ξ -	Δx_5 kft	x_5 kft	
.2	29.6	209.6	.106	3.99	23.0	
.4	59.1	239.1	.228	8.58	27.6	
.6	88.7	268.7	.368	13.84	32.8	
.8	118.3	298.3	.554	20.82	39.8	
1.0	147.8	327.8	1.000	37.6	56.6	← Apogee

Given: $g = 32.15\ ft/sec^2$, $\gamma = 1.400$, $A = 0.1364$ sq.ft., $K = 0.920$

$V_o = 5.400$ kfps, $\theta_o = 85.00^o$

Case 2: $W = 180.0$ lbs, $m = \frac{180.0}{32.15} = 5.60$ slugs, $\frac{K\gamma A}{2m} = \frac{0.920\ (1.400)0.1364}{2(5.60)} = 15.70\ (10)^{-3}\ ft^3/lb\text{-}sec^2$

Choose: $h_1 = 10$ kft., $h_2 = 30$ kft., $h_3 = 60$ kft., $h_4 = 180$ kft.

Column	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Source	Given	Given	Col.14	Eq.3.7	Tables	Tables	Eq.3.6	Eq.3.13	Tables	Fig. 5	Eq.3.8	Eq.3.12	Eq.3.11	Eq.3.12	Eq.3.14	Col.15	Eq.3.15	Col.17
n	h_n	h_n	V_n	Δh_n	θ_n	$\sin\theta_n$	$\cos\theta_n$	$\overline{\sin\theta_n}$	$\overline{\theta_n}$	ΔI_n	ΔV_{D_n}	$\frac{1}{2}\Delta \mathcal{V}_{D_n}$	$\Delta \mathcal{V}_n$	ΔV_n	ΔX_n	X_n	Δt_n	t_n
-	kft	kft	kfps	-	o	-	-	-	o	kip-sec/ft^2	kfps	-	-	kfps	kft	kft	secs	secs
0	0	-	5400	-	85.00	.9962	.0872	-	-	-	-	-	-	-	-	0	-	0
1	10	10	5.084	.0220	84.94	.9961	.0881	.99615	84.97	16.2	.255	.0236	-.0585	-.316	.880	.88	1.92	1.92
2	30	20	4.651	.0497	84.81	.9959	.0904	.9960	84.88	19.1	-.301	.0296	-.0852	-.433	1.796	2.68	4.13	6.05
3	60	30	4.261	.0891	84.57	.9955	.0946	.9957	84.68	11.0	.174	.0187	-.0839	-.390	2.79	5.47	6.77	12.82
4	180	120	3.168	.425	82.82	.9922	.1250	.9938	83.62	3.4	.054	.0063	-.2565	-1.093	13.42	18.89	32.54	45.36

For h > 180 kft., Figure 7 is used. Here $\Delta h_5 = \frac{(V_4 \sin\theta_4)^2}{2g}\eta = \frac{(3.168\ [.9922]\ )^2}{.0643}\eta = 153.9\ \eta$

$$\Delta t_5 = \frac{V_4 \sin\theta_4}{g}\xi = \frac{3.168(.9922)}{.03215}\xi = 97.9\ \xi,\ \Delta X_5 = \frac{V_4^2 \sin\theta_4 \cos\theta_4}{g} = \frac{(3.168)^2 .9922\ (.1250)}{.03215}\xi = 38.8\ \xi$$

For velocity, Eq.(3.3) is used. Here $V_5 = \sqrt{V_4^2 - 2g\Delta h_5} = \sqrt{(3.168)^2 - .0643\Delta h_5}$

η	Δh_5	h_5	ξ	ΔX_5	X_5	V_5	Δt_5	t_5	
-	kft	kft	-	kft	kft	kfps	secs	secs	
.2	30.8	210.8	.106	4.11	23.0	2.84	10.4	55.8	
.4	61.6	241.6	.228	8.85	27.7	2.47	22.3	67.7	
.6	92.3	272.3	.368	14.28	33.2	2.03	36.0	81.4	
.8	123.2	303.2	.554	21.5	40.4	1.46	54.2	99.6	
.9	138.4	318.4	.682	26.5	45.4		66.8	112.2	
1.0	153.9	333.9	1.000	38.8	57.7	.40	97.9	143.3	← Apogee

Case 3: Data as for Case 1: Choose h_1 = 100 kft.

Column	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Source	Given	Given	Col.14	Eq.3.7	Tables	Tables	Eq.3.6	Eq.3.13	Tables	Fig. 5	Eq.3.8	Eq.3.12	Eq.3.11	Eq.3.12	Eq.3.14	Col.15
n	h_n	Δh_n	V_n	Δk_n	θ_n	$\sin\theta_n$	$\cos\theta_n$	$\overline{\sin\theta_n}$	$\overline{\theta_n}$	ΔI_n	ΔV_{D_n}	$\frac{1}{2}\Delta v_{D_n}$	Δv_n	ΔV_n	ΔX_n	X_n
-	kft	kft	kfps	-	o	-	-	-	o	kip-sec/ft^2	kfps	-	-	kfps	kft	kft
0	0	-	5.400	-	85.00	.9962	.0872	-	-	-	-	-	-	-	-	0
1	100	100	3.888	.2205	84.33	.9951	.0988	.9956	84.63	49.1	.819	.0758	-.2800	- 1.512	9.39	9.39

Case 4: Data as for Case 2: Choose h_1 = 100 kft.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Cols. 15 & 16	Cols. 17 & 18
0	0	-	5.400	-	85.00	.9962	.0872	-	-	-	-	-	-	-	-	-
1	100	100	3.937	.2205	84.33	.9951	.0988	.9956	84.63	49.1	.776	.0718	-.2710	-1.463	9.39	21.54

For h > 100 kft, Figure 7 is used for Trajectory and Eq.(3.3) for Velocity:

Case 3: $\Delta h_2 = \frac{(V_1 \sin\theta_1)^2}{2g}\eta = \frac{(3.888\,[.9951])^2}{.0843}\eta = 233.4\eta$, $\Delta X_2 = \frac{V_1^2 \sin\theta_1 \cos\theta_1}{g}\xi = \frac{(3.888)^2 .9951(.0988)}{.03215}\xi = 46.2\xi$

Case 4: $\Delta h_2 = \frac{(3.937\,[.9951])^2}{.0643}\eta = 239.2\eta$, $\Delta X_2 = \frac{(3.937)^2 .9951(.0988)}{.03215}\xi = 47.4\xi$

$V_2 = \sqrt{V_1^2 - 2g\Delta h_2} = \sqrt{(3.937)^2 - .0643\Delta h_2}$, $\Delta t_2 = \frac{V_1 \sin\theta_1}{g}\xi = \frac{3.937(.9951)}{.03215}\xi = 122.0\xi$

		Case 3				Case 4					Case 4		
η	ξ	Δh_2	h_2	ΔX_2	X_2	Δh_2	h_2	ΔX_2	X_2	V_2	Δt_2	t_2	
-	-	kft	kft	kft	kft	kft	kft	kft	kft	kfps	sec	sec	
.2	.106	46.6	146.6	4.9	14.3	47.8	147.8	5.0	14.4	3.52	12.9	34.5	
.4	.228	93.3	193.3	10.5	19.9	95.6	195.6	10.8	20.2	3.06	27.8	49.4	
.6	.368	140.0	240.0	17.0	26.4	143.6	243.6	17.5	26.9	2.50	44.9	66.4	
.8	.554	186.7	286.7	25.6	35.0	191.3	291.3	26.3	35.7	1.79	67.6	89.1	
.9	.682	210.0	310.0	31.5	40.9	215.4	315.4	32.4	41.8	1.29	83.2	104.7	
1.0	1.000	233.4	333.4	46.2	55.6	239.2	339.2	47.4	56.8	.36	122.0	143.5	←Apogee

APPENDIX C

Projectile Trajectory - Present Method

$B = 4000 \text{ psf}, \frac{K \ell A}{2m} = 22.50 \quad \frac{M}{B} = 22.50 \quad \frac{4.4}{4000} = 24.75\ (10)^{-3}\ \text{ft}^3/\text{lb-sec}^2$

Column	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Source	Given	Given	Col.14	Eq.3.7	Slide Rule	Slide Rule	Eq.3.6	Eq.3.13	Slide Rule	Fig. 5	Eq.3.8	Eq.3.12	Eq.3.11	Eq.3.12	Eq.3.14	Col.15	Eq.3.15	Col.17
n	h_n	Δh_n	v_n	$\Delta \bar{h}_n$	θ_n	$\sin\theta_n$	$\cos\theta_n$	$\overline{\sin\theta}_n$	$\bar{\theta}_n$	ΔI_n	ΔV_{D_n}	$\frac{1}{2}\Delta V_{D_n}$	$\Delta \bar{v}_n$	ΔV_n	ΔX_n	X_n	Δt_n	t_n
-	kft	kft	kfps	-	o	-	-	-	o	kip-sec/ft^2	kfps	-	-	kfps	kft	kft	secs	secs
0	0	--	4.700	--	48.0	.744	.669	--	--	--	--	--	--	--	--	0	--	0
1	10	10	4.085	.0291	47.3	.735	.679	.7395	47.6	16.2	.542	.0576	-.1308	-.615	9.1	9.1	3.08	3.08
2	20	10	3.624	.0386	46.2	.722	.692	.7285	46.7	11.1	.377	.0462	-.1129	-.461	9.4	18.5	3.57	6.65
3	40	20	2.954	.0979	43.3	.686	.729	.704	44.7	13.5	.475	.0655	-.1849	-.670	20.2	38.7	8.64	15.29
4	80	40	2.128	.2955	29.7	.496	.868	.591	36.25	7.6	.313	.0538	-.2793	-.826	54.5	93.2	26.65	41.94
5	80	0	2.128	0	-29.7	-.496	.868	--	--	--	--	--	--	--	121.2	214.4	65.70	107.64
6	40	-40	2.386	-.567	-46.1	-.721	.693	-.6085	-37.4	7.6	.309	.0727	+.121	+.253	52.3	266.7	29.15	136.79
7	20	-20	2.219	-.226	-51.3	-.780	.626	-.7505	-48.6	13.5	-.445	.0933	-.070	-.167	17.6	284.3	11.58	148.37
8	10	-10	2.024	-.1305	-53.9	-.808	.589	-.794	-52.55	11.1	.346	.0780	-.0878	-.195	7.7	292.0	5.94	154.31
9	0	-10	1.708	-.1572	-56.8	-.836	.548	-.822	-55.35	16.2	.488	.1206	-.1560	-.316	6.9	298.9	6.52	160.83

Δh @ Apogee $= \frac{(V_4 \sin \theta_4)^2}{2g} = \frac{(2.128(.496))^2}{.0643} = \frac{1.056^2}{.0643} = 17.3$ kft. $\therefore h_{ap} = 97.3$ kft. (+4.2%)

ΔX @ Apogee $= \frac{V_4^2 \sin \theta_4 \cos \theta_4}{g} = \frac{(2.128)^2 .496(.868)}{.03215} = 60.6$ kft. $\therefore X_{ap} = 153.8$ kft. (+1.1%)

Δ @ Apogee $= \frac{V_4 \sin\theta_4}{g} = \frac{1.056}{.03215} = 32.85$ secs. $\therefore t_{ap} = 74.8$ sec. (+3.3%)

V @ Apogee $= V_4 \cos \theta_4 = 2.128\ (.868) = 1.848$ kfps. (-3.5%)

$X_{impact} = 298.9$ kft. (+1.9%)

$t_{impact} = 160.8$ sec. (+4.1%)

$V_{impact} = 1.708$ kfps (-9.0%)

APPENDIX C

Projectile Trajectory - HARP Computer Program

VEHICLE DESCRIPTION
LONG RANGE DELIVERY

INITIAL WEIGHT NOMINAL....	581.	LBS
GUN ELEVATION.............	48.00	DEGREES
MUZZLE VELOCITY...........	4700.	FT. PER SEC.

DRAG COEFFICIENTS

	Z	P	C0	C1	C2	C3
SUBSONIC..........	0.3600	0.1000				
SUPERSONIC(CURVE).		0.1000	0.7570	-0.2226	0.0295	-0.0015

WEIGHT OF FUEL.........	0.00	LBS
MASS FLOW..............	0.00	LBS/SEC.
BODY DIAMETER..........	11.00	IN.
NOZZLE EXIT DIAMETER...	-0.00	IN.
SPECIFIC IMPULSE.......	0.00	SECS

DELAY BEFORE IGNITION...........	0.00	SECS
DELAY BEFORE BURNOUT............	0.00	SECS

COMMENTS	TIME SECS	VELOCITY FT./SEC.	RANGE FEET	ALTITUDE FEET	ALTITUDE KILOMETERS	CD	MACHNO	ATTITUDE DEGREES	DYN PRESS LBS/SQ.FT
LAUNCH	0.0	4700.	0.	0.	0.000	0.235	4.2077	48.000	2[illegible]242.919
	20.0	2765.	48742.	48476.	14.776	0.328	2.8557	40.201	1484.682
	40.0	2236.	89292.	76523.	23.325	0.383	2.3102	27.950	253.663
	60.0	19[illegible]2.	128311.	90935.	27.717	0.418	2.0016	11.551	98.804
APOGEE	72.4	1915.	152155.	93389.	28.465	0.427	1.9343	-0.043	82.740
	80.0	1[illegible]20.	166668.	92473.	28.186	0.425	1.9434	-7.259	85.896
	100.0	2065.	204472.	81356.	24.798	0.403	2.1329	-24.905	170.615
	120.0	2311.	241296.	57990.	17.676	0.374	2.3867	-39.033	551.837
	140.0	2325.	274852.	24763.	7.548	0.385	2.2893	-50.934	2998.681
IMPACT	154.5	1878.	293227.	-127.	-0.039	0.459	1.6806	-57.417	4211.251

$$4000 = \frac{581(144)}{C_D \, \pi \, (5.5)^2} = \frac{880}{C_D}$$

$C_D = 0.22$ @ $M = 4.4$

APPENDIX D

Trajectory Equations for Vehicle under Rocket Thrust with Zero Drag, Constant g

Newton's 2nd Law for Directions Parallel and Normal to Trajectory:

$$\text{Parallel:} \quad F - mg \sin \theta = m \frac{dV}{dt} \qquad \text{D-1}$$

$$\text{Normal:} \quad - mg \cos \theta = mV \frac{d\theta}{dt} \qquad \text{D-2}$$

Elimate V from D-1 and D-2 by dividing D-2 by $\frac{d\theta}{dt}$ and differentiating with respect to t:

$$m \frac{dV}{dt} = - \frac{d}{dt} \left(\frac{mg \cos \theta}{d\theta/dt} \right) = mg \left\{ \sin \theta + \cos \theta \frac{d^2\theta/dt^2}{(d\theta/dt)^2} \right\} = F - mg \sin \theta$$

Therefore, on simplifying the above, the differential equation for $\theta(t)$ is obtained:

$$\frac{d^2\theta}{dt^2} + (2 \tan \theta - \alpha \sec \theta) \left(\frac{d\theta}{dt}\right)^2 = 0 \qquad (4.1)$$

Solution for α = constant

Set $\omega = \frac{d\theta}{dt}$ Then Eq. (4.1) can be written

$$\frac{d\omega}{\omega} + (2 \tan \theta - \alpha \sec \theta) \, d\theta = 0 \qquad \text{D-3}$$

This can be integrated directly to give

$$\frac{\omega}{\omega_{n-1}} = \frac{\cos^{\alpha-2} \theta_{n-1} (1 + \sin \theta)^{\alpha}}{\cos^{\alpha-2} \theta (1 + \sin \theta_{n-1})^{\alpha}} \qquad \text{D-4}$$

This is integrated in turn to give, after some simplification:

$$\int_{\theta_{n-1}}^{\theta_n} \frac{\cos^{\alpha-2} \theta \, d\theta}{(1 + \sin \theta)^{\alpha}} = - \frac{\cos^{\alpha-1} \theta_{n-1} \Delta \tau_n}{(1 + \sin \theta_{n-1})^{\alpha}} \qquad (4.2)$$

Solution for Δh_n for α = constant

Divide D-1 by m and differentiate with respect to t:

$$\frac{d^2V}{dt^2} = - g \cos\theta \frac{d\theta}{dt} = \frac{g^2 \cos^2\theta}{V} \qquad \text{D-5}$$

using D-2. Again, if D-1 is solved for $g \sin\theta$:

$$g \sin\theta = g\alpha - \frac{dV}{dt} \qquad \text{D-6}$$

or

$$g^2(1 - \cos^2\theta) = g^2\alpha^2 - 2g\alpha \frac{dV}{dt} + \left(\frac{dV}{dt}\right)^2 \qquad \text{D-7}$$

Equating $g^2 \cos^2\theta$ from D-5 and D-7, θ is eliminated:

$$V \frac{d^2V}{dt^2} = - g^2(\alpha^2 - 1) + 2g\alpha \frac{dV}{dt} - \left(\frac{dV}{dt}\right)^2$$

or

$$\frac{d}{dt}\left(V \frac{dV}{dt}\right) - 2g\alpha \frac{dV}{dt} = - g^2 (\alpha^2 - 1) \qquad \text{D-8}$$

D-8 can be integrated directly to give:

$$V \left\{\frac{dV}{dt} - 2g\alpha\right\} = V_{n-1} \left\{\frac{dV}{dt}\Big/_{n-1} - 2g\alpha\right\} - g^2(\alpha^2 - 1)(t - t_{n-1}) \qquad \text{D-9}$$

$\frac{dV}{dt}$ can be eliminated, using D-6:

$$V \left\{\alpha + \sin\theta\right\} = V_{n-1}\left\{\alpha + \sin\theta_{n-1}\right\} + g(\alpha^2 - 1)(t - t_{n-1})$$

or

$$V = V_{n-1} \left\{\frac{\alpha + \sin\theta_{n-1}}{\alpha + \sin\theta}\right\} + \frac{g(\alpha^2 - 1)(t - t_{n-1})}{(\alpha + \sin\theta)} = \frac{dh}{dt} \frac{1}{\sin\theta} \qquad \text{D-10}$$

This gives directly

$$dh = V_{n-1} (\alpha + \sin\theta_{n-1}) \frac{\sin\theta \, dt}{(\alpha + \sin\theta)} + g(\alpha^2 - 1) \frac{\sin\theta \, (t - t_{n-1})dt}{(\alpha + \sin\theta)} \qquad \text{D-11}$$

D-11 is made non-dimensional by multiplication with $\frac{2g}{V_{n-1}^2}$ and then integrated:

$$\Delta h_n = 2(\alpha + \sin\theta_{n-1}) \int_0^{\Delta\tau_n} \frac{\sin\theta \, d\tau}{(\alpha + \sin\theta)} + 2(\alpha^2 - 1) \int_0^{\Delta\tau_n} \frac{\sin\theta \, \Delta\tau \, d\tau}{(\alpha + \sin\theta)}$$

(4.6)

APPENDIX E MARTLET 4 TRAJECTORY CALCULATIONS

Comparison With HARP Case 1046
Parts I, II, III - Constant-g Calculations from Launch to Burn-Out of Stage 2

% Errors from HARP Case 1046	V_n	X_n	h_n	t_n	θ_n
Ignition Stage 1	-.29	-1.34	-	+.410	-.42
BurnOut Stage 1	+.45	-1.14	+3.41	+.20	+6.46
BurnOut Stage 2	+1.18	-1.67	+5.16	+.15	+7.69

I. Pre-Ignition Climb: $V_o = 6.000$ kfps, $\theta_o = 33.0^o$, $g_o = .03215$ kfps^2, K = 1.00, $\gamma = 1.40$, A = 1.478 ft^2

$m_o g_o = 2735$ lbs, $\frac{K\gamma A}{2m_o} = \frac{1.00(0.70)1.478}{2735}(32.15) = 12.15\,(10)^{-3}$ ft^3/lb-sec^2

II. 1st Stage Burning: $h_3 = 40.0$ kft, $V_{j4} = 9.000$ kfps, $g_o b_4 = \frac{1600}{15} = 106.7$ lbs/sec., $C_4 \sim .04808$/kft., $(p/a)_3 = 4.45(10)^{-4}$ $\frac{\text{lb-sec}}{\text{ft}^2\text{kft}}$

III. 2nd Stage Burning: Start at t_4, $V_{j5} = 9.320$ kfps, $g_o b_5 = \frac{400}{10} = 40.0$ lbs/sec., $g_o m_4 = 775$ lbs, $(p/a)_4 = 0$

I. Pre-Ignition Climb

Col.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Source	Given	Given	Col.14	Eq.3.7	Slide Rule	Slide Rule	Eq.3.6	Eq.3.13	Slide Rule	Fig.5	Eq.3.8	Eq.3.12	Eq.3.11	Eq.3.12	Eq.3.14	Col.15	Eq.3.15	Col.17
n	h_n	Δh_n	v_n	Δk_n	θ_n	$\sin\theta_n$	$\cos\theta_n$	$\overline{\sin\theta_n}$	$\overline{\theta_n}$	ΔI_n	Δv_{D_n}	$\frac{1}{2}\Delta v_{D_n}$	$\Delta\sigma_n$	Δv_n	ΔX_n	X_n	Δt_n	t_n
-	kft	kft	kfps	-	o	-	-	-	o	kip-sec/ft^2	kfps	-	-	kfps	kft	kft	sec	sec
0	0	-	6.000	-	33.0	.544	.839	-	-	-	-	-	-	-	-	0	-	0
1	10	10	5.579	.0179	32.1	.531	.846	.538	32.5	16.2	.366	.0305	-.0703	-.421	15.7	15.7	3.21	3.21
2	20	10	5.263	.0207	31.2	.519	.855	.525	31.65	11.1	.257	.0230	-.0567	-.316	16.2	31.9	3.51	6.72
3	40	20	4.808	.0464	28.8	.482	.875	.5005	30.1	13.5	.328	.0311	-.0864	-.455	34.5	66.4	7.95	14.67

II and III: 1st and 2nd Stage Burning

Col.	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
Source	Given	Col.19or17	Col.3or45	Col.19	Eq.4.13	Fig.14	Eq.4.25	Eq.4.27	Eq.4.24	Eq.4.26	Eq.4.7	Eq.4.5	Slide Rule	Slide Rule	Eq.3.13
n	Δt_n	t_n	v_{j_n}	$g_o\Delta m_n$	ν_n	ΔH_n	ε_{n-1}	δ_n	Δk_n	$\overline{\alpha}_n$	$f(\overline{\alpha}_n)$	$\cos\theta_n$	θ_n	$\sin\theta_n$	$\overline{\sin\theta_n}$
-	sec	sec	-	lbs	-	-	-	-	-	-	-	-	o	-	-
4	15.00	29.67	1.872	1600	.415	.530	5.84	.0796	.1558	18.63	.1437	.886	27.7	.465	.465 .473
								.0811	.1586	18.63	.1437	.886	27.7	.465	.4735
5	10.00	39.67	.764	400	.484	.838	19.60	.0234	.0299	22.95	.1064	.888	27.3	.459	.460 .462
								.0236	.0301	22.95	.1064	.888	27.3	.459	.462

Col.	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
Source	Col.27	Col.34	Eq.4.20	Eq.4.26	Eq.4.19	Eq.4.19	Eq.4.22	Eq.4.22	Col.41-Col.40	Eq.4.23	Eq.3.12	Col44	Col.33	Eq.3.14	Col.47
n	Δh_n	h_n	$C-\zeta_n$	ζ_n	$\epsilon^{-\zeta_n \Delta h_n}$	ΔJ_n	Δv_{D_n}	Δv_{F_n}	$\Delta v_{F_n}-\Delta v_{D_n}$	Δv_n	Δv_n	v_n	$\overline{\theta}_n$	ΔX_n	X_n
-	kft	kft	/kft	/kft	-	kip-sec/ft^2	-	-	-	-	kfps	kfps	o	kft	kft
4	57.1	97.1	.01470	.03338	.1488	11.4	.061	1.645	1.584	1.538	7.40	12.21	28.25	106.4	172.8
5	69.8	166.9	-	-	-	0	0	.555	.555	.543	6.63	18.84	27.55	134.0	306.8

HARP TRAJECTORY PROGRAM - R. H. MCKEE - MK 3 10 AUG 66 CASE 1046 PAGE 1

CASE NUMBER	1046	MUZZLE VELOCITY	[illegible]300.
NO. OF STAGES	3	ELEVATION	35.00
PAYLOAD WEIGHT	75.00	AZIMUTH	119.23
AREA	1.478	LONGITUDE	-59.48
STEP SIZE	5.	LATITUDE	13.07
OUTPUT INTERVAL	100.	HEIGHT	150.

STAGE NUMBER	1	2	3
IGNITION INDICATOR	0	3	1
FIRING INDICATOR	1	1	0
WEIGHT (TOTAL)	2735.0	775.0	275.0
AREA	1.478	1.478	1.478
EXHAUST AREA	0.785	0.785	0.785
BURNING TIME	15.00	10.00	5.00
IGNITION VALUE	40.00	-0.00	0.30
FIRING ELEVATION	-0.00	-0.00	-0.00
FIRING AZIMUTH	-0.00	-0.00	-0.00
MASS FRACTION	0.82	0.80	0.80
SPECIFIC IMPULSE	280.	290.	290.

TIME	HEIGHT	RANGE	VELOCITY	AIRSPEED	PATH ANGLE	ELEVATION	AZIM	WIND EL	WIND AZ	DYN PRES	PATH AZ
0.00	0.02	0.00	7173.	6030.	27.10	33.00	6.35	33.00	6.35	42597.17	118.23
STAGE 1 IGNITION - LONGITUDE		59.317	LATITUDE	12.984							
14.61	6.58	11.07	6050.	4822.	22.67	28.92	7.30	28.92	7.30	6829.67	118.33
29.61	15.44	28.76	13372.	12155.	23.50	26.02	3.34	26.02	3.34	3278.24	118.45
STAGE 1 BURNOUT - LONGITUDE		-59.057	LATITUDE	12.847							
STAGE 2 IGNITION - LONGITUDE		-59.057	LATITUDE	12.847							
29.61	15.44	28.76	13372.	12155.	23.50	26.02	3.34	26.02	3.34	3278.24	118.45
39.61	26.09	51.26	19637.	18621.	25.70	25.35	2.27	25.35	2.27	426.04	118.56
STAGE 2 BURNOUT - LONGITUDE		-58.726	LATITUDE	12.672							
139.61	144.00	319.68	18660.	17370.	19.99	21.55	2.54	21.55	2.5	0.00	119.70
239.61	235.36	569.81	17770.	16415.	15.84	17.19	2.76	17.19	2.76	0.00	120.59
339.61	303.88	806.83	17120.	15713.	11.32	12.35	2.93	12.35	2.93	0.00	121.25
439.61	347.06	1034.89	16704.	15261.	6.51	7.13	3.05	7.13	3.05	0.00	121.72
539.61	366.28	1257.53	16518.	15059.	1.53	1.68	3.11	1.68	3.11	0.00	122.00
STAGE 3 IGNITION - LONGITUDE		-40.565	LATITUDE	1.984							
564.13	367.35	1311.65	16508.	15048.	0.30	0.30	0.00	0.33	3.12	0.00	122.04
569.13	367.42	1325.22	24649.	23181.	0.24	0.55	0.00	0.26	2.03	0.00	120.96
STAGE 3 BURNOUT - LONGITUDE		-40.369	LATITUDE	1.865							
	PERIGEE	349.56	APOGEE	381.97							

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13. ABSTRACT

An approximate numerical method for calculation of trajectories of hypersonic vehicles is described, with particular reference to gun-launched vehicles. Because of the form of the hypersonic drag coefficient, a relatively simple calculation using functions which depend only on altitude is possible. Reasonable accuracy can be obtained even for computations in which very large calculation intervals are used. The method is particularly suitable for preliminary engineering calculations which do not justify a detailed computer solution, and has the added advantage of giving the analytic forms linking input and output. The technique is shown to be applicable to glide trajectories with and without drag and to rocket powered trajectories as well.

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